

H. O. No. 147

Maneuvering Board Manual

ARMED FORCES NAVY COMMISSIONED

OFFICERS AND ENLISTED



V
245
.m3
1941

DATA LIBRARY ARCHIVES

Wood's Hole Oceanographic Institution



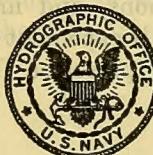
0 0301 0072507 3

H. O. No. 217

Maneuvering Board Manual

UNITED STATES NAVY DEPARTMENT

HYDROGRAPHIC OFFICE



UNITED STATES GOVERNMENT PRINTING OFFICE
WASHINGTON : 1941

Hydrographic Board

STATUTES OF AUTHORIZATION

There shall be a Hydrographic Office attached to the Bureau of Navigation in the Navy Department for the improvement of the means for navigating safely the vessels of the Navy and of the mercantile marine by providing, under the authority of the Secretary of the Navy, accurate and cheap nautical charts, sailing directions, navigators, and manuals of instructions for the use of all vessels of the United States, and for the benefit and use of navigators generally (R. S. 431).

The Secretary of the Navy is authorized to cause to be prepared, at the Hydrographic Office attached to the Bureau of Navigation in the Navy Department, maps, charts, and nautical books relating to and required in navigation, and to publish and furnish them to navigators at the cost of printing and paper, and to purchase the plates and copyrights of such existing maps, charts, navigators, sailing directions, and instructions as he may consider necessary, and when he may deem it expedient to do so, and under such regulations and instructions as he may prescribe (R. S. 432).

PREFACE

The Maneuvering Board Manual is compiled primarily for the purpose of furnishing officers of the U. S. Navy with a printed publication, in an unrestricted status, to which ready reference can be made for aid in the solution of tactical problems. Explanations have, therefore, been simplified and reduced to a minimum consistent with clarity.

From time to time, in the past, many different typewritten pamphlets on this subject have been compiled by various Naval officers and much material has also appeared in scattered articles in the Naval Institute Proceedings, but no regular printed publication on this subject has ever been issued. This text is therefore arranged with the assistance of officers attached to the Post-graduate School, United States Naval Academy, Annapolis, Md. The material has been freely drawn from all of the above-mentioned sources, but principally from the problems as given in the Maneuvering Board Manual prepared for students and graduates of the Postgraduate School.

Constructive criticism is therefore freely invited of any part of the text or its drawings.

G. S. BRYAN

*Captain, U. S. Navy,
Hydrographer.*

11

INTRODUCTION

The ability of certain outstanding navigators and tacticians to rapidly and efficiently carry out missions, conduct scouting and search operations, and shift stations within a fleet or other mobile unit has long been known. Although their skill has been described by such terms as "having developed a good seaman's eye," basically their aptitude has been the result of being able to apply the principles of relative movement to the particular problem at hand. Relative movement is an everyday phenomenon. The most familiar example of this is the apparent movement of celestial bodies across the sky. As the globe turns from the West to the East, to an observer stationed on the earth, the celestial bodies appear to rise in the East and set in the West. When two trains on adjacent tracks are moving in the same direction but at different speeds, to passengers on the faster train it appears that the slower train is moving backwards. By movement relative to the faster train and ignoring the actual direction and distance traveled over the face of the earth by both, that is what the slower train is doing.

The essential difference between the relative movement method of solving problems and the usual navigational plot method, is one of origins. The latter uses a point fixed with respect to the earth and called a "Chart Point." The travel of units, portrayed by lines on the chart used, represents directions and distances actually traversed on the face of the earth or over the ground. Such a diagram, when used in this publication, will be referred to as the "Navigational Plot." The lines representing the travel of units over the ground in this diagram are called "Chart Lines." When several units are being plotted on this diagram, their exact positions for any particular time must be carefully delineated before their positions relative to each other can be found. For a composite picture of the actions of several units, this is excellent; for planning actions in advance, the amount of trial and error involved usually causes much delay, so the relative movement method is to be preferred in most cases.

The relative movement method uses a moving unit instead of a chart point as the point of origin. The unit so chosen, designated as the "Guide," remains fixed in the Relative Plot, although it represents a ship moving over the earth. The

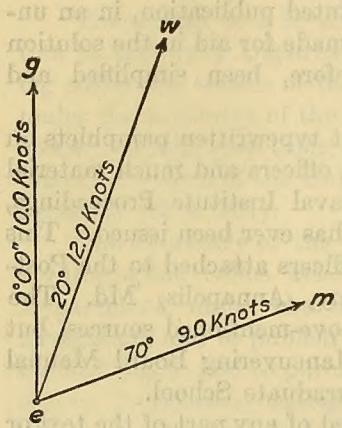


FIGURE 1.

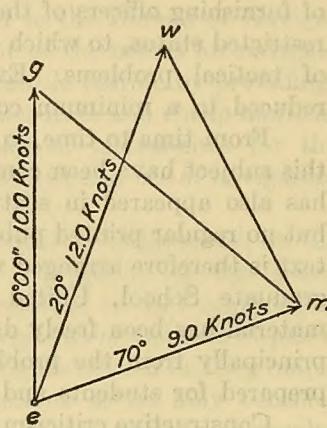


FIGURE 2.

movement of all other units concerned in the problem is referred to this guide and their travel is portrayed by "Relative Movement Lines." The relative movement line of a maneuvering unit, with respect to the guide, is defined by drawing a line through plotted successive positions of the maneuvering unit, as determined by range finder and compass aboard the guide. These positions are laid off from a fixed point representing the position of the guide in the Relative Plot. The Maneuvering Board has been designed to facilitate this plotting as well as to show graphically all the maneuvers involved.

In the relative movement method, two diagrams are normally required, the "Vector Diagram" and the "Relative Plot." The "Navigational Plot" may be added as part of the solution or as a check on results. The Vector Diagram is so called because every line in it is a vector and therefore indicates both magnitude and direction. The magnitude of the vector is its length, which applied to the scale in use indicates a velocity, or rate. The direction of the vector is shown by its inclination, an arrow being added to the head of the vector to prevent reciprocal errors. Vector quantities are added and subtracted geometrically by the Parallelogram Law as outlined in standard textbooks.

The point of origin in a vector diagram is usually a point fixed with respect to the earth, and for convenience is lettered "e." Thus, vectors originating at "e" show direction and rate of travel with respect to the earth, or over the ground.

In Figure 1, three such vectors are shown. $e \dots g$ indicates the travel of a unit G , making 10.0 knots in direction 000° over the ground; $e \dots w$ represents the travel of the wind in direction towards 020° at 12 knots over the ground; while $e \dots m$ shows the unit M on course 070° at a speed of 9.0 knots over the ground. The Wind Vector indicates the direction *toward* which the wind is blowing, instead of the usual method of describing the direction *from* which it originates. These vectors, or any others based on the same units of motion, may be combined to form the Vector Diagram.

Figure 2 shows the combination of the above vectors to indicate concurrent travel. By joining $g \dots m$ another vector is formed. If we consider m as the head and g as the foot of the resultant vector, $g \dots m$ gives the direction and rate of travel of unit M relative to unit G . In the corresponding Relative Plot, G would be stationary while M traversed a line parallel to $g \dots m$ and at the rate indicated by the length of $g \dots m$. If, on the contrary, we consider g as the head and m the foot of this vector, then $m \dots g$ gives the direction and rate of travel of unit G relative to unit M , and in the corresponding Relative Plot, M would remain stationary while G traversed a line parallel to and in direction $m \dots g$, at the rate

indicated by the length of $m \dots g$. Similarly, joining w and m , and considering m the head and w the foot of the vector so formed, $w \dots m$ represents the travel of unit M in respect to the wind. If this vector is reversed and w made the head, then $m \dots w$ represents the travel of the wind in respect to unit M or shows the apparent wind on M .

From the above it will be noted that all lines in the Vector Diagram are vectors, which have in them only the quantities of *direction* and *rate of travel*. No other quantities can be directly obtained from this diagram.

The Relative Plot is the second diagram employed in the Relative Movement Method. This diagram has a fixed point, representing the unit used as a guide, and such Relative Lines as the solution may produce. These Relative Lines show *direction* and *distance travelled, both relative to the Guide Unit*. The Relative Plot, therefore, has only the quantities of *direction* and *distance*.

The common factor in both the Vector Diagram and the Relative Plot is *direction*. Hence, the interchange of data from one diagram to the other must be based upon this common quantity. The element of Time, required in practically all problems is the factor by which speed, from the Vector Diagram, is converted to Distance in the Relative Plot or vice versa. The solution for Time alone may be reached by dividing the distance, from the Relative Plot, by the speed, from the Vector Diagram. The two are interchanged through their common quantity of direction.

Although the majority of the problems dealt with herein are solved by the Relative Movement Method, recourse is fully had to other methods which more readily yield the desired results. Particular mention must be made of the Navigational Plot, which can always be utilized as a proof of the solution, regardless of what method was actually used in working the problem. A solution by this method may be extremely laborious, involving trial and error, but once the results are obtained by some easier and quicker method, it is a matter of but a few moments to obtain a check by completing the Navigational Plot. In actual practice, whenever time permits, it should be employed not only to check the results, but also to insure that there is no navigational hazard in the direction of your own ship's motion.

It must be pointed out that the mere working out of a problem on the Maneuvering Board is not enough. The progress of your own vessel along the line of relative movement must be checked by both bearings and rangefinder ranges. In some instances the guide may change both course and speed considerably without previous signal. It is far easier to set up another problem based on your instantaneous position than to try to involve the previous solution with the problem presented by the new conditions. All officers should be familiar with the simplest types of problems and their rapid solution. The operator at sea, however, should never become so intent on figuring out his next proper move on a Maneuvering Board that he fails to keep a sharp lookout for incidental ships, such as other maneuvering units, merchant ships, or aircraft carriers, about to launch or land aircraft, which may happen to wander into his set-up. Also, no shoals or menaces to navigation are shown on the Maneuvering Board. A lack of alertness in this regard will never be compensated by the ability to find perfect solutions on a Maneuvering Board.

SUGGESTIONS

The Maneuvering Board, HO 2665, is especially designed to facilitate the solution of problems by Vector Diagrams and Relative Movement Plots, although any chart equipped with a compass rose may be used if dividers, parallel rulers, and a proper scale are available.

When using the Maneuvering Board, place the Vector origin, e , at the center of the diagram.

Letter the head and foot of each vector as it is drawn and indicate the head by an arrowhead. Leave the foot plain or enclosed in a small circle. Use small letters in the Vector Diagram to correspond with capital letters designating the same unit in the Relative Plot. Any letters desired by the operator may be used to indicate the various units concerned. In this publication, G is used to designate the Guide unit and g indicates the head of the Guide's vector. If a single maneuvering unit is involved, it is usually designated M , with the corresponding m as its vector head, but this lettering may be varied for the sake of clarity.

In the Vector Diagram, it is suggested that the letters e , g , and w be used as herein. In the Relative Plot, indicating the unit Guide by G will tend to prevent errors.

In the figures illustrating the various examples in this publication, distinctive lines are used to emphasize the construction of the set-up. For ordinary solution of problems, this is not necessary.

Courses and bearings should be drawn in their true directions. Relative bearings are easily found by orienting the diagram. The application of variation and deviation to true courses and bearings after the solution is reached readily yields the magnetic or compass courses and bearings, with less chance for error than if the conversion were attempted earlier in the solution.

No set rule can be given for the scale to be used in the Relative Plot. It should be as large as the size of the board will permit. G may be placed at any point, but usually it is more convenient to place it at the center of the board, superimposed upon e . Care should be taken that the Relative Plot and the Vector Diagram are not confused thereby. Sometimes, by placing G elsewhere, a much larger scale can be employed, with consequent increase in accuracy.

Care should be exercised to differentiate between the scale used in the Vector Diagram and the scale used in the Relative Plot. These diagrams are entirely separate. The habit of noting in the upper right-hand corner of the board the proper scales used, such as 1 division = 2 miles, or 1 division = 3 knots, will generally prevent confusion in this matter. Also, when using HO 2665, if the scales on the side are employed, a D over the distance scale used and an S over the speed scale employed, will further tend to reduce error. Confusing the distance scale and the speed scale is the most frequent source of mistake made by the beginner. Finally, the scales chosen should be as large as practicable.

Strive for accuracy but not by the expenditure of excessive time. It is much more advantageous for the officer conning to have an approximate course and speed to start with, modified as necessary later for the exact course and speed required, than to be slow in changing station. The standard of accuracy required should be sufficiently exact as to produce results which

would be acceptable at sea. Use the correct values for your own course and speed and the best estimated values for the guide's course and speed, which is not often exactly known.

When in formations, cultivate the habit of having the position of all units plotted on the Maneuvering Board, with the formation guide at the center. This removes doubt as to the location of those units which may interfere with the expeditious changing of your own station as well as permits a more rapid solution of the problem presented. The value of this habit, especially when on the scouting line, cannot be overestimated. A rough sketch on plain paper may assist in choosing the proper scale for plotting the formation.

The use of accurate parallel rulers and a sharp pencil will aid in neatness and clarity. If, however, no parallel rulers are available, two drafting triangles will suffice.

When determining the point of tangency between a line and a circle, estimating by eye is insufficient. Determine the direction of the tangent, add or subtract 90° and lay off this direction from the center of the circle. Its intersection with the original line will determine the point of tangency.

Follow the latest practice of using three-figure numbers for courses or bearings. If a decimal part of an hour or mile is to be indicated, place a zero before the decimal point. All courses or bearings are true unless otherwise indicated.

Finally, remember that while the maneuvering unit travels along the Relative Movement Line, its heading coincides with this line only in the special case in which the course of the maneuvering unit is the same as the guide's course. In other words, the maneuvering unit generally travels along the Relative Movement Line "crab fashion."

VI

SCOUTING LINE

The Maneuvering Board, HO page 1, is designed to facilitate the plotting of formations and formations of units in the Relative Movement Line, especially when combining with a maneuvering unit or a formation.

Upon using the Maneuvering Board, place the A scale on the left, the B scale on the right, and the C scale on the center. The latter is the base line for the formation. The A scale is used to lay off the distance between the formation and the formation guide. The B scale is used to lay off the distance between the formation and the formation guide. The C scale is used to lay off the distance between the formation and the formation guide.

After the formation has been plotted, the formation guide is plotted on the C scale. The distance between the formation and the formation guide is then plotted on the B scale. The distance between the formation and the formation guide is then plotted on the C scale.

After the formation has been plotted, the formation guide is plotted on the C scale. The distance between the formation and the formation guide is then plotted on the B scale. The distance between the formation and the formation guide is then plotted on the C scale.

After the formation has been plotted, the formation guide is plotted on the C scale. The distance between the formation and the formation guide is then plotted on the B scale. The distance between the formation and the formation guide is then plotted on the C scale.

After the formation has been plotted, the formation guide is plotted on the C scale. The distance between the formation and the formation guide is then plotted on the B scale. The distance between the formation and the formation guide is then plotted on the C scale.

After the formation has been plotted, the formation guide is plotted on the C scale. The distance between the formation and the formation guide is then plotted on the B scale. The distance between the formation and the formation guide is then plotted on the C scale.

GLOSSARY

The following definitions of terms used in this publication are listed for the information of students:

Air Course.—The corrected compass course steered by aircraft through the air. By combining the Wind Vector and the known Air Speed, the equivalent Ground Course and Speed are obtained.

Air Speed.—The speed of an aircraft relative to the air. This speed, when known, is represented by a circle of proper radius to the scale in use with its center at w in the Vector Diagram. This circle is treated in the same manner as ground speed circles drawn with e as their center and used for surface craft. Equivalent Ground Speed is obtained by combining the Air Speed with the Wind Vector.

Chart Course.—The true direction over the surface of the earth that an aircraft or ship is intended to travel.

Chart Point.—A properly located point which is fixed in respect to the ground or earth.

Chart Track.—A line representing the true course and distance made good or desired to be made good between stations on the earth.

Current Vector.—A vector with origin at e , representing travel of the water over the ground or current. The head of this vector is usually lettered c . It is seldom necessary to use this vector except when anchoring.

Constant Bearing Line.—The relative line traversed by a unit whose bearing from the reference unit does not change. This condition exists when the guide unit and the maneuvering unit arrive at a common point, diverge from a common point, or when the two have the same vector. In the first case, this is called the "Collision Bearing."

Guide.—The chosen unit to which movement of other units concerned is referred. In the Relative Plot, the Guide remains stationary.

Maneuvering Unit.—Any moving unit set up in the problem except the Guide. Specifically, a Maneuvering Unit is one whose movements are under investigation.

Navigational Plot.—A diagram such as is used in ordinary chart or navigational work. Sometimes the Navigational Plot is used as the alternate solution which may be more rapid than the Relative Movement Plot. Its most common usage is to check or prove the results found in the latter.

Relative Movement Line.—A line passing through successive relative positions occupied by the Maneuvering Unit plotted from bearings and range-finder distances taken on the Guide, whose position remains stationary in the Relative Plot. It is commonly called a "Relative Line."

Relative Plot.—One of the fundamental diagrams used in Relative Movement methods. A fixed point indicates the position of the Guide and one or more Relative Movement Lines are properly related to this point for the movements of the Maneuvering Unit. This diagram yields position, as well as direction and distance of relative travel.

Relative Position.—The position occupied by one unit with respect to another, defined by range and bearing of one from the other. The bearing may be either true or relative.

Slope.—A direction or inclination. This term is usually used in connection with the transfer of a line parallel to itself from one diagram to another.

Speed Circle.—A circle drawn about a point or the end of a vector with the radius equal to a given speed. Such circles drawn with e as center indicate speed over the ground. If drawn with w as center they indicate speed of aircraft through the air.

Time Line.—A line joining the heads of two vectors which represent successive courses and speeds of a specific unit in passing from an initial to a final position in known time, via a specified intermediate point. This line also touches the head of a constructive unit which proceeds directly from the initial to the final position in the same time. By general usage this constructive unit is called the Fictitious Ship, and the head of its vector, marked " f ," divides the Time Line into segments inversely proportional to the times spent by the Reference Unit on the first and second legs. The Time Line is used in two-course problems.

Vector.—A straight line which indicates by its inclination and length respectively the direction and rate of travel of a unit represented by the head of the vector relative to a unit represented by the foot of the vector.

Vector Diagram.—A coordinated set of vectors representing concurrent travel of the units concerned. It is a fundamental diagram in the Relative Movement Method. This diagram yields rate and direction of travel.

SECTION I

SINGLE VECTOR CASES

The most usual problem facing the Maneuvering Board operator is when there is a single vector for the Guide and a single vector for the Maneuvering Unit. In this section are typical cases based upon variations of this premise. For this reason, the set-up of the problem and the method of solution should be carefully studied.

In some instances a choice between two or more Maneuvering Unit Vectors is possible. This arises from the statement of the problem being sufficiently broad, either by accident or design, to permit more than one solution. In actual practice, other factors may be present which will preclude an alternate solution.

In the first few examples quotation marks are included for the sake of emphasizing steps. These are omitted in the later examples.

Case I

TO PROCEED FROM ONE RELATIVE POSITION TO ANOTHER IN A SPECIFIED TIME

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL AND FINAL RELATIVE POSITIONS, AND TIME ALLOWED.

TO DETERMINE: COURSE AND SPEED OF MANEUVERING UNIT.

Example.—Guide “G” is on course 205° true, speed 10.0 knots. The Maneuvering Unit “M”, now 12.0 miles due west of “G”, is ordered to take station 20.0 miles dead ahead of “G”, arriving on station in 1.5 hours.

Required.—(a) Course for “M”. (b) Speed of “M”. (See fig. 3).

Procedure.—Plot Guide at “G” and locate initial and final positions of “M” at M_1 and M_2 respectively. Join $M_1 \dots M_2$. Draw vector “e . . . g”, representing course and speed of Guide.

Since the Maneuvering Unit moves along the Relative Movement Line $M_1 \dots M_2$, transfer the slope of this line to “g”. From “g”, in the same direction that M_2 lies from M_1 , measure a vector equal to the rate of Relative Travel, which is equal to the Relative Distance $M_1 \dots M_2$ divided by the time available, 1.5 hours. This is found most readily by using the Logarithmic Scale at the bottom of the Maneuvering Board, placing a straight edge along the Relative Distance (18.5 miles) and the time allotted (1.5 hours or 90 minutes). This Relative Speed is found to be 12.4 knots, which is the length of “g . . . m”. Vector “e . . . m” represents the required course and speed of the Maneuvering Unit.

Answer.—(a) 185° . (b) 21.4 knots.

NOTE.—Had the Maneuvering Unit in this problem been a plane, there would have been no change in the Relative Plot. In the Vector Diagram, however, the Wind Vector, “e . . . w” would be laid off from “e” if it measured the true wind; if it were apparent wind on “G” it would be laid off as “g . . . w” from “g”. The Air Course and Air Speed would be found by joining “m” and “w” instead of “m” and “e” as shown. The true course to be steered would be found by transferring the Slope “w . . . m” to the compass rose on the diagram. This is illustrated more fully in some of the following examples.

Successive positions of the Maneuvering Unit are shown in the Relative Plot of figure 3 in order to stress the important fact that although the maneuvering Unit is travelling down the Relative Movement Line, its heading does not necessarily coincide with the slope of this line. It is, however, inclined in the direction shown by the vector “e . . . m”.

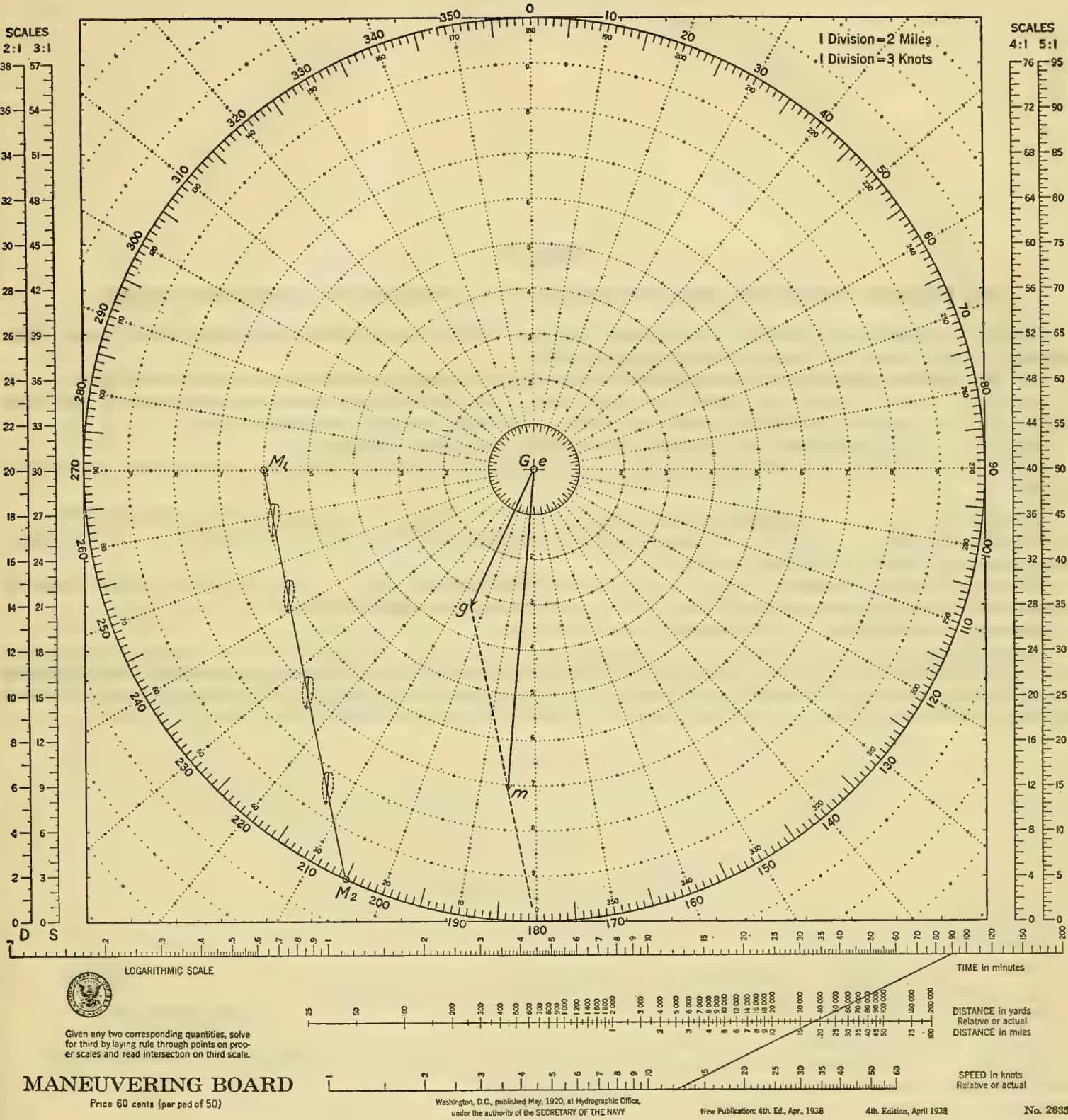


FIGURE 3.

Case II

TO PROCEED FROM ONE RELATIVE POSITION TO ANOTHER ON A SPECIFIED COURSE

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL AND FINAL RELATIVE POSITIONS, AND THE COURSE TO BE USED BY THE MANEUVERING UNIT.

TO DETERMINE: SPEED OF MANEUVERING UNIT AND TIME TO ARRIVE IN FINAL POSITION.

Example.—Guide on course 230° , speed 10.0 knots. Ship " M ", now 4.0 miles on the starboard beam of the Guide is ordered to take station 10.0 miles broad on the port quarter of the Guide, steering course 180° en route.

Required.—(a) Speed of " M ". (b) Time for " M " to reach new position. (See fig. 4.)

Procedure.—Plot Guide at " G " and locate initial and final positions of Maneuvering Unit at M_1 and M_2 , respectively. Join $M_1 \dots M_2$.

Draw vector " $e \dots g$ ", representing course and speed of " G ". From " e " lay out course line specified for " M ".

Transfer slope " $M_1 \dots M_2$ " to " g ", intersecting the course line of " M " at " m ". The vector " $e \dots m$ " represents the course and speed of the Maneuvering Unit.

Divide the Relative Distance. " $M_1 \dots M_2$ ", by the Relative Speed, " $g \dots m$ ", to obtain the time required to reach the new station on the specified course. This is easily done on the Logarithmic Scale.

Answer.—(a) 8.82 knots. (b) 98 minutes or 1 hour 38 minutes.

NOTE.—Should the Maneuvering Unit be a plane, the wind vector, " $e \dots w$ ", would appear in the Vector Diagram. The specified ground course is laid off from " e ", as above. By reference to " w " the Air Speed and Air Course corresponding could be found. If the Air Course were specified, this direction would be laid off from " w " instead of from " e ", and the proper Air Speed determined by the intersection of this course line with the transferred slope " $M_1 \dots M_2$ " laid off from " g ". Time in either event would be obtained in the manner illustrated.

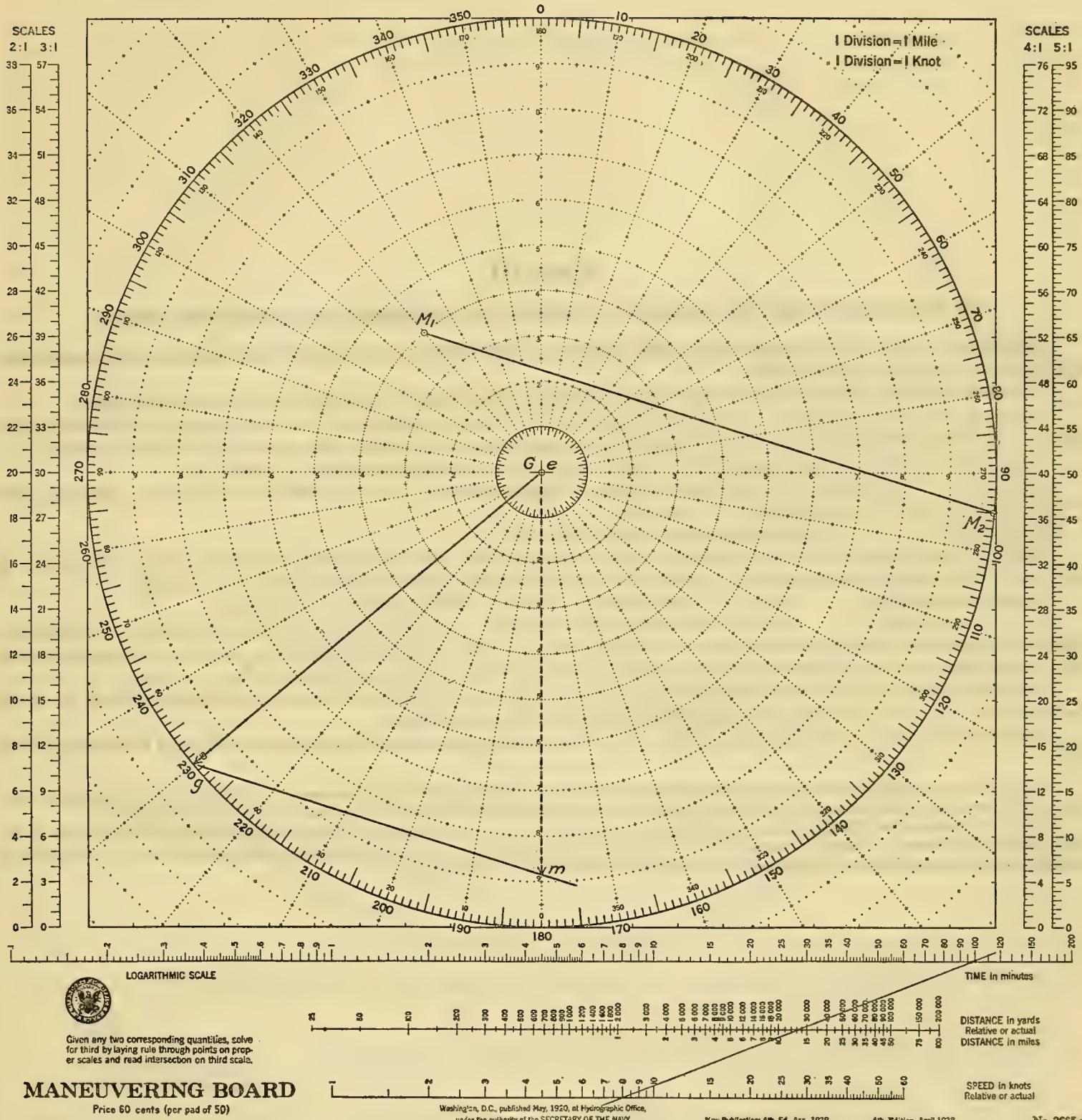


FIGURE 4.

Case III

TO PROCEED FROM ONE RELATIVE POSITION TO ANOTHER AT A SPECIFIED SPEED

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL AND FINAL RELATIVE POSITIONS, AND SPEED TO BE USED BY MANEUVERING UNIT.

TO DETERMINE: COURSE OF MANEUVERING UNIT AND TIME TO ARRIVE IN FINAL POSITION.

Example.—Guide on course 037° , speed 16.0 knots. Ship " M ", now bearing 060° (true) and distant 11.5 miles from the Guide, is ordered to take position 16.0 miles 30° abaft the starboard beam of the Guide, using a speed of 12.0 knots enroute.

Required.—(a) Course or courses for " M ". (b) Time required to reach final position. (See fig. 5.)

Procedure.—Plot Guide at " G ", and locate initial and final positions of " M " at M_1 and M_2 respectively. M_2 bears 120° relative from " G " or 157° (true) from " G ". Join $M_1 \dots M_2$.

Draw vector " $e \dots g$ ", representing course and speed of " G ".

With " e " as center and with radius equal to the specified speed for " M " (12.0 knots), describe a circle. This circle, of course, will contain all the courses available to " M " at the specified speed, no matter what the other requirements.

Transfer the slope $M_1 \dots M_2$ to " g ", cutting the speed circle of " M " at points " m_1 " and " m_2 ".

" $e \dots m_1$ " and " $e \dots m_2$ " are vectors that " M " could use. An inspection of the diagram quickly reveals the fact that the latter vector yields the greatest Relative Speed; therefore " M " would naturally choose the course indicated by this vector unless there were other vessels or obstacles present which would dictate the use of the vector " $e \dots m_1$ ".

The time required to arrive at the final position is found by dividing the Relative Distance by the Relative Speed as usual. For vector " $e \dots m_2$ " and using the Logarithmic Scale this is shown graphically.

Answer.—(a) $153\frac{1}{2}^\circ$ or alternate course 047° . (b) 52 minutes by most expeditious course or 4.43 hours by steering course 047° .

NOTE.—Only one solution can be obtained if the speed of the Maneuvering Unit is greater than the speed of the Guide. Two solutions are otherwise possible unless the specified speed for the Maneuvering Unit is the minimum speed that would allow him to reach the final position.

In case the Maneuvering Unit is a plane, draw the wind vector " $e \dots w$ ", and about " w " draw the speed circle with radius equal to the given air speed of the plane. This speed circle is treated in precisely the same manner as the ground speed circle, drawn about " e " in figure 5. Since the air course would be desired in this case, it is obtained by reference to " w " instead of " e ".

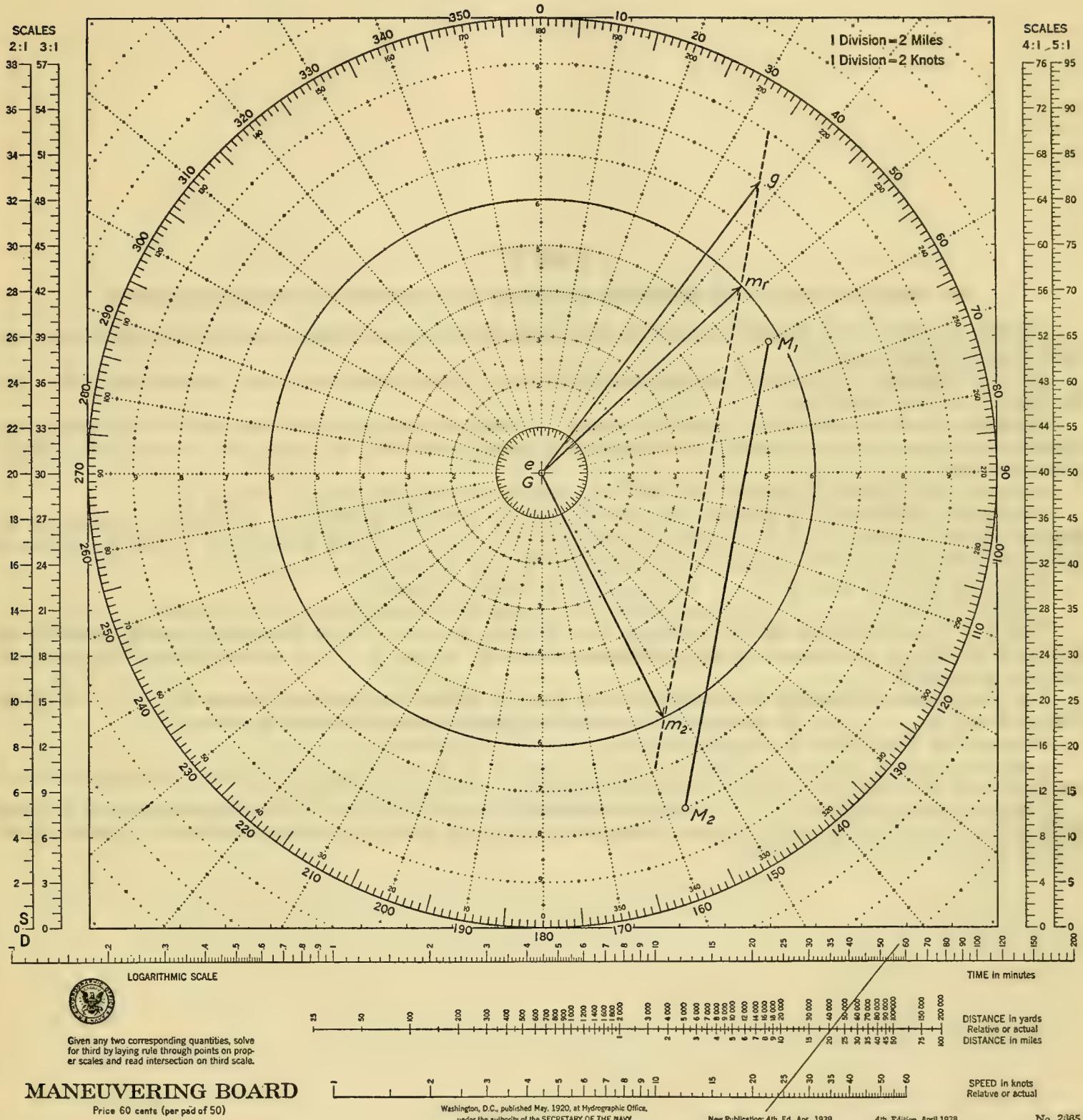


FIGURE 5.

Case IV

TO PROCEED FROM ONE RELATIVE POSITION TO ANOTHER AT MINIMUM SPEED

GIVEN: COURSE AND SPEED OF GUIDE AND THE INITIAL AND FINAL RELATIVE POSITIONS FOR THE MANEUVERING UNIT.

TO DETERMINE: COURSE AT MINIMUM SPEED AND TIME REQUIRED FOR MANEUVERING UNIT TO REACH FINAL POSITION.

Example.—Guide on course 148° at speed 15.0 knots. A vessel, M , now located 600 yards dead astern of the Guide, receives orders to open out to a distance of 19,000 yards and a bearing of 280° from the Guide, using minimum speed enroute to conserve fuel.

Required.—(a) Course and speed for M . (b) Time required to reach final position. (See fig. 6.)

Procedure.—Plot Guide at G and the initial and final positions of M at M_1 and M_2 . In this example, realizing that the Maneuvering Board is 20 divisions wide, a more accurate result is obtained by plotting G and M_2 on the final line of bearing and separated by 19 units. M_1 is located 600 yards bearing 328° from G . Join $M_1 \dots M_2$.

From point e , lay out vector $e \dots g$ in direction 148° , length 15.0 knots.

Transfer the slope $M_1 \dots M_2$ to g , extending it in the same direction from g that M_2 lies from M_1 .

From e drop a perpendicular on this transferred slope, intersecting at m . This is most readily done by observing the direction of the slope on the compass rose and subtracting or adding 90° to this. $e \dots m$ is the vector of the course and minimum speed for M .

The time required to complete this evolution may be found by dividing the Relative Distance $M_1 \dots M_2$ by the Relative Speed $g \dots m$. By using the Logarithmic Scale, this time may be obtained in minutes.

Answer.—(a) Course 189° at minimum speed 11.4 knots. (b) 57 minutes.

NOTE.—The foot of the perpendicular, m , must fall on the same side of g that M_2 lies in respect to M_1 . Should it fall on the other side of g , it indicates that M must use a speed greater than that of the Guide and the solution becomes indeterminate.

For a plane, draw the wind vector, $e \dots w$, and drop the perpendicular from w instead of from e . The corresponding air course is obtained by reference to w instead of e . Note that the course is always normal to the line of Relative Movement and can therefore be obtained without plotting if the direction of the latter is known.

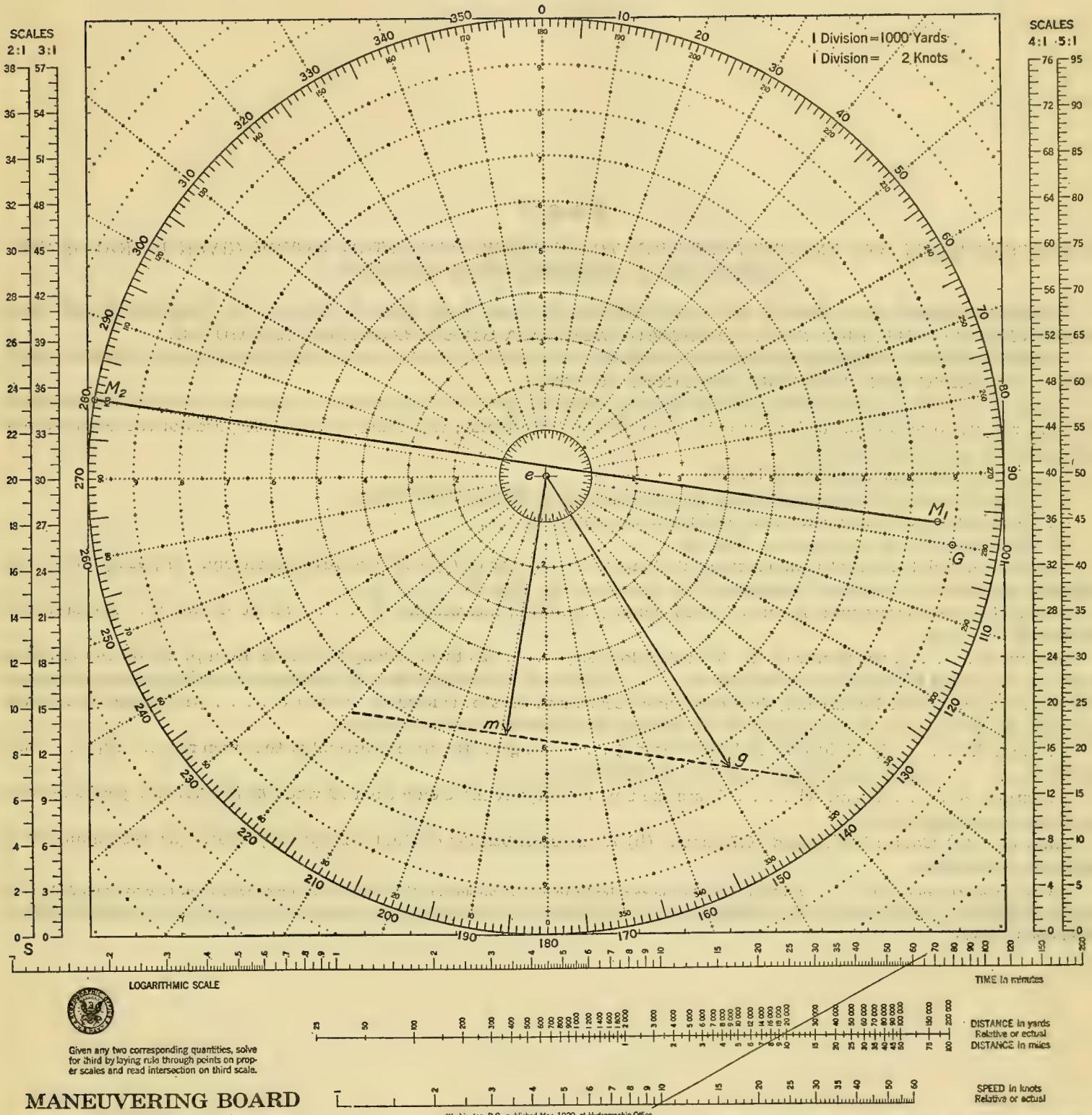


FIGURE 6

Case V

TO PROCEED FROM ONE RELATIVE POSITION TO ANOTHER, REMAINING WITHIN GIVEN RANGE OF THE GUIDE FOR A SPECIFIED TIME ENROUTE

GIVEN: COURSE AND SPEED OF THE GUIDE, INITIAL AND FINAL RELATIVE POSITIONS OF THE MANEUVERING UNIT, AND TIME TO REMAIN WITHIN A GIVEN RANGE OF THE GUIDE.

TO DETERMINE: COURSE AND SPEED FOR MANEUVERING UNIT AND TRUE BEARINGS FROM GUIDE WHEN ENTERING AND LEAVING REQUIRED RANGE.

Example.—Guide on course 200° , speed 15.0 knots. Ship M , now 35° on the starboard quarter of the Guide and distant 8.0 miles, receives orders take station 8.0 miles dead ahead of the Guide, remaining within 6,000 yards of the Guide for 45 minutes for the purpose of signalling.

Required.—(a) Course and speed for M .

- (b) Time to come within required range and true bearing of Guide at that range.
- (c) True bearing of Guide when passing beyond required range.
- (d) Time to arrive at new station. (See fig. 7.)

Procedure.—Plot Guide at G and locate initial and final positions of M at M_1 and M_2 respectively. M_1 , being 35° on the starboard quarter of G , bears 145° relative or 345° true from G . Join $M_1 \dots M_2$.

About G draw a circle with radius of 6,000 yards (3.0 miles), intersecting $M_1 \dots M_2$ at K and K^1 . Measure the distance $K \dots K^1$.

Lay out $e \dots g$, the vector of G . Transfer slope $M_1 \dots M_2$ to g . Along this slope, from g , lay off the Relative Speed which is equal to the Relative Distance $K \dots K^1$ divided by the time that M is to remain within the specified range. This is most readily done by utilizing the Logarithmic Scale, and thus m is located. Vector $e \dots m$ indicates the course and speed for M to reach its new station while fulfilling the required conditions enroute.

Distance $M_1 \dots K$, divided by Relative Speed $g \dots m$ gives the time of arrival at the given range. $M_1 \dots M_2$, divided by $g \dots m$ gives the time to arrive on final station.

Directions $K \dots G$ and $K^1 \dots G$ are the true bearings of the Guide from M when M reaches and passes beyond the given range respectively.

Answer.—(a) Course 196° , speed 19.7 knots. (b) 73 minutes, bearing of Guide $129\frac{1}{2}^\circ$. (c) 056° . (d) 190 minutes or 3 hours and 10 minutes.

NOTE.—Had the slope $M_1 \dots M_2$ been outside the required range, the obvious action indicated would have been to close to 6,000 yards in the most expeditious manner, take up the course and speed of the Guide for 45 minutes, and then proceed to the new station by another Manoeuvring Board set-up.

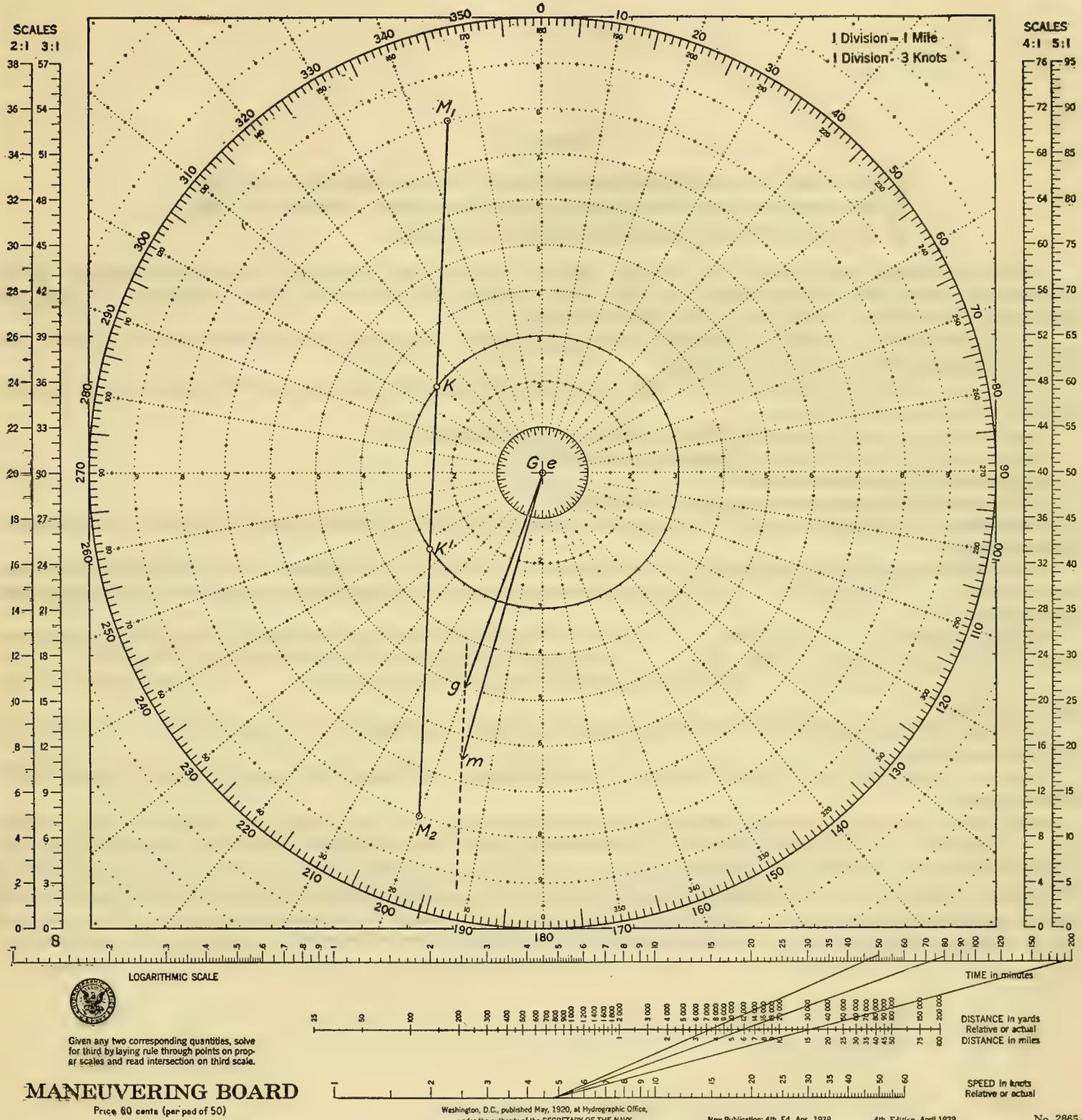


FIGURE 7.

Case VI

- (1) TO PASS AT SPECIFIED RANGE FROM GUIDE (SHOWN IN FIG. 8)
- (2) TO PASS OUTSIDE SPECIFIED RANGE OF GUIDE (SHOWN IN FIG. 9)
- (3) TO PASS WITHIN SPECIFIED RANGE OF GUIDE

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL RELATIVE POSITION OF MANEUVERING UNIT, COURSE OR SPEED REQUIREMENTS (IF ANY), AND RANGE (1) AT WHICH IT IS DESIRED TO PASS, (2) OUTSIDE OF WHICH TO PASS, OR (3) TO PASS WITHIN.

TO DETERMINE: LIMITING COURSE, LIMITING SPEED, OR BOTH, FOR MANEUVERING UNIT AND TIME OF REACHING MINIMUM RANGE.

Example A.—Guide on course 335° speed 18.0 knots. Ship M , now 18.0 miles bearing 340° from the Guide, wishes to pass the latter at a range of 8.0 miles.

Required.—(a) Course or courses of M at 12.0 knots if crossing ahead of G .
(b) Course(s) of M at 12.0 knots if passing to eastward of G .
(c) Speed of M if course used is 295° .
(d) Course(s) of M using minimum speed. (See fig. 8.)

Procedure.—Plot Guide at any point G , and initial position of M at M_1 . About G draw a circle with radius equal to the given range.

From M_1 draw tangents to circle about G , establishing points K and K' . M may either travel down the Relative Movement Lines $M_1 \dots K$ or $M_1 \dots K'$, based upon either crossing ahead of and passing to westward of G or else passing to eastward of G .

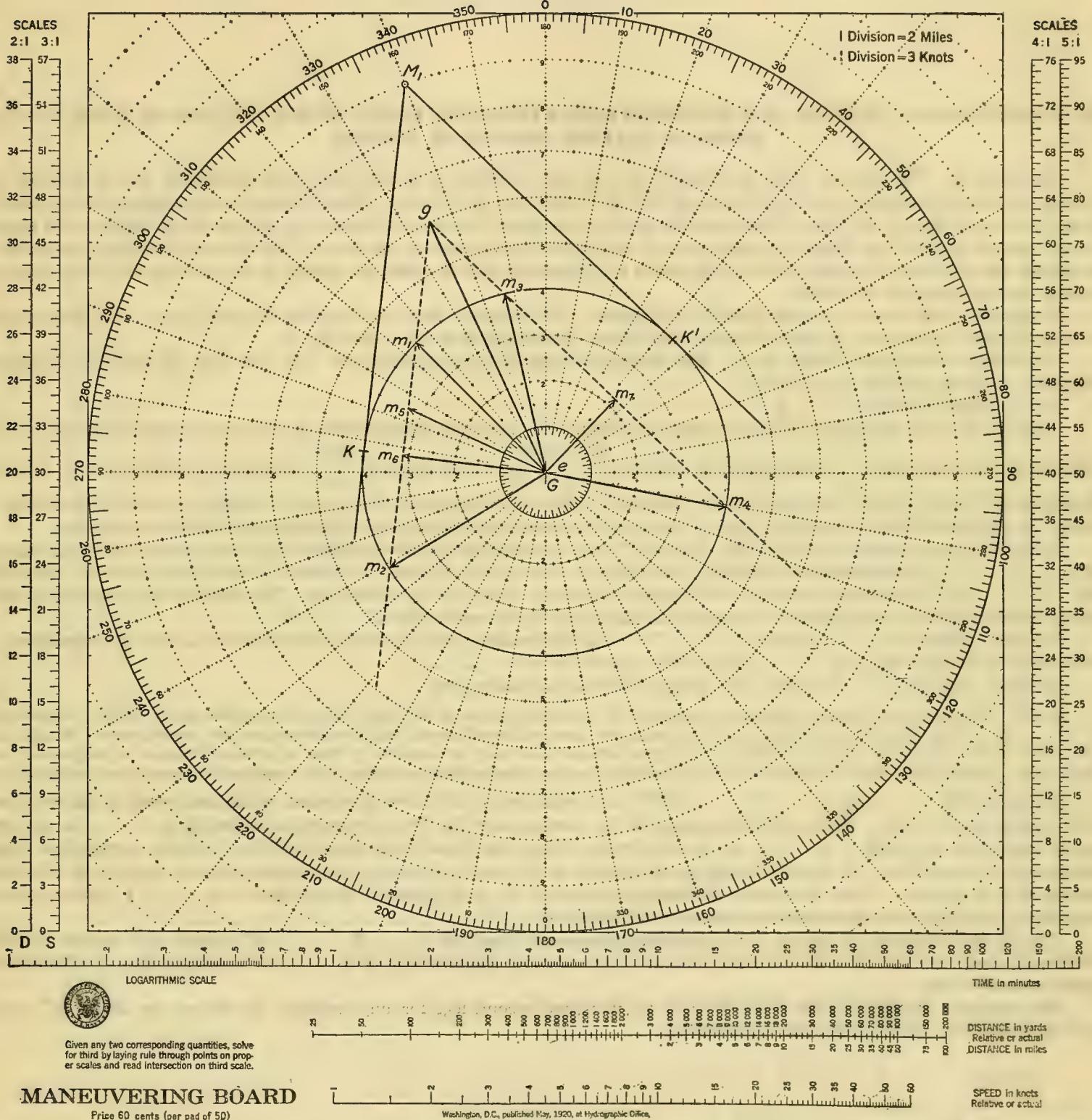
Transfer slopes $M_1 \dots K$ and $M_1 \dots K'$ to g , cutting M 's 12.0-knot speed circle at m_1 and m_2 for the former slope and at m_3 and m_4 for the latter. Slope $M_1 \dots K$ also cuts the projected 295° course line at m_5 . Vectors $e \dots m_1$, $e \dots m_2$, $e \dots m_3$, or $e \dots m_4$ can be used by M at the given speed. Vector $e \dots m_5$ indicates M 's speed on the given course.

To obtain minimum speeds and the courses corresponding, drop perpendiculars $e \dots m_6$ and $e \dots m_7$ from e on the transferred slopes.

Answer.—(a) 315° or 238° . (b) 347° or $100\frac{1}{2}^\circ$. (c) 10.0 knots. (d) Course $276\frac{1}{2}^\circ$ speed 9.5 knots or course $043\frac{1}{2}^\circ$ speed 6.5 knots.

NOTE.—The vectors found above will cause M to pass G at exactly the given range. If it were desired to pass within the given range, the slopes $M_1 \dots K$ and $M_1 \dots K'$ would be drawn so as to pass within the limiting range circle about G , and the same procedure followed. If it is desired to pass outside of the given range, the slopes $M_1 \dots K$ and $M_1 \dots K'$ and inclined so as to neither cut nor touch the limiting range circle, followed by the procedure mentioned above. A special application of this latter problem is given in example B on the following pages.

It will be noted that where the speed of M is less than that of the Guide, two solutions will be found for each transferred slope, unless the transferred slopes are tangent to the speed circles, when only one solution is possible for each slope. In case the transferred slopes neither cut nor touch the speed circles, no solution is possible. The conditions imposed by the statement of the problem should indicate which of the resultant vectors to use.



TO INSURE PASSING OUTSIDE OF A SPECIFIED RANGE FROM THE GUIDE WITH A COLUMN OF SHIPS, USING HEAD OF COLUMN CHANGE OF COURSE

Example B.—The leading vessel and flagship of a Cruiser Division of 3 ships in column formation, with a distance of 1,000 yards between vessels, is on Fleet Course of 135° at Fleet Speed 10.0 knots. Cruiser Flagship, now stationed 8,000 yards broad on the port bow of the Fleet Guide, receives orders to proceed with the division to new station 8,000 yards on the starboard quarter of the Fleet Guide, keeping clear of the antisubmarine screen en route. The Cruiser Division Commander decides to use 20.0 knots, to allow 3,000 yards radius for the screen, and to make the change to new station by column movements, passing ahead of the Guide.

Required.—(a) Course to cross ahead of the Guide. (b) Course to final station after clearing Guide. (c) Range and bearing of Fleet Guide from Cruiser Flagship when course is changed as in (b). (See fig. 9.)

Procedure.—Plot Fleet Guide at G . Plot initial positions of the cruisers at CF , CA , and CB , CF being the Cruiser Flagship, and final position of CF at CF' .

Draw Guide's vector $e \dots g$.

Advance G along the Fleet Course the distance it would run while CB at 20.0 knots in advancing to the point where CF changed to the first course. This distance is equal to $\frac{\text{Speed of } G}{\text{Speed of } CB} \times 2,000 \text{ yards} = \frac{10.0}{20.0} \times 2,000 \text{ yards} = 1,000 \text{ yards}$. This is done because in any column movement speed changes are made simultaneously while course changes are made in succession. About the advanced Guide's position G' draw a circle with radius equal to 3,000 yards, enclosing the antisubmarine area.

From CF draw a tangent to this circle. From CF' draw a second tangent to this offset circle, intersecting the first tangent at X . $CF \dots X$ represents the Relative Movement of the Cruiser Flagship in respect to the Guide while on the first leg. $X \dots CF'$ represents similarly the Relative Movement while on the second leg. The bearing and distance of the Guide from CF at the turning point for starting the second leg is represented by $X \dots G$.

Transfer slopes $CF \dots X$ and $X \dots CF'$ to g , intersecting the 20.0-knot speed circle at c_1 and c_2 respectively. The vector of the first leg is $e \dots c_1$ and of the second leg is $e \dots c_2$.

Answer.—(a) $211\frac{1}{2}^\circ$. (b) 285° . (c) Range 4,200 yards, bearing 348° .

NOTE.—This particular example is included to emphasize the fact that a column of ships may occupy considerable space. When maneuvering in the vicinity of other formations and units, all course and speed changes planned should take this entire space into consideration. Should the total length of the column be neglected, vessels following the column leader may be compelled to take an echelon formation to keep out of the danger area. This further reacts to increase the speed required to resume proper station in column, with consequent increase in fuel consumption.

The path $CB \dots CA \dots Y \dots CB''$, etc., indicates the Relative Movement of the last vessel in the column with respect to the Guide. While CB is advancing to the turning point for CF , its vector is indicated by $e \dots cb$, and its speed relative to the Guide is $g \dots cb$, or 10.0 knots. Using the Logarithmic Scale and the 3 minutes required for this advance, it is found that CB advances 1,000 yards relative to G , making it occupy the original relative position of CA , the second ship in column. From this point, its movement relative to G is indicated by the slope $CA \dots Y$, which passes outside of the 3,000-yard antisubmarine screen circle about G .

The Cruiser Flagship, as shown by its Relative Movement Lines $CF \dots X \dots CF'$, passes well outside of this 3,000-yard circle, but the extra distance travelled has prevented the embarrassing of the other ships in column and maintained better concentration.

The positions occupied by the three ships on the first and second legs for any moment are shown by CF'' , CA'' , and CB'' and by CF''' , CA''' , and CB''' respectively.

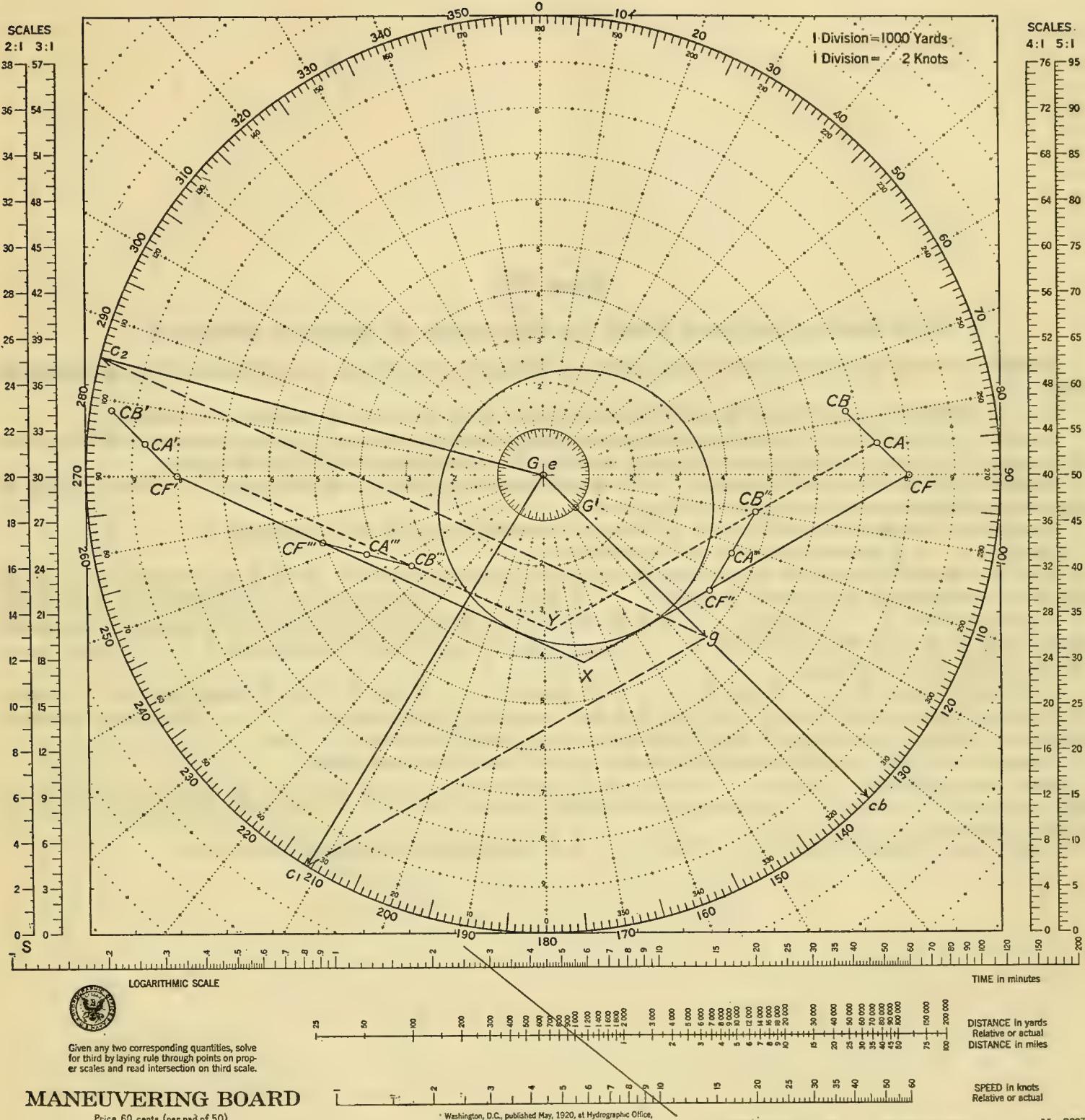


FIGURE 9.

Case VII

WITH SPEED LESS THAN GUIDE, TO PASS CLEAR AT MAXIMUM DISTANCE

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL RELATIVE POSITION AND SPEED OF MANEUVERING UNIT.

TO DETERMINE: COURSE OF MANEUVERING UNIT AND MINIMUM RANGE ATTAINED.

Example.—A submarine with maximum submerged speed of 8.0 knots wishes to keep clear as far as possible from a cruiser now bearing 135° distant 12.0 miles, whose course and speed have been determined as 345° and 24.0 knots.

Required.—(a) Best course for submarine. (b) Minimum range and time to reach this range. (c) Relative bearing of cruiser from submarine at minimum range. (See fig. 10.)

Procedure.—Plot the cruiser as Guide at G. Locate the present position of the submarine at S.

Lay out $e \dots g$, representing course and speed of Guide.

About e draw an 8.0-knot circle representing the maximum submerged speed of S. From g draw tangents to this speed circle, points of tangency being s_1 and s_2 . Vectors $e \dots s_1$ and $e \dots s_2$ represent the two courses open to S at 8.0 knots.

Transfer the slopes $g \dots s_1$ and $g \dots s_2$ to S. Although the Relative Speed is the same for either course it is readily apparent that the Relative Movement Line $S \dots X$ passes farther away from G than the line $S \dots Y$. $e \dots s_1$ is the best course for the submarine.

Draw perpendiculars $G \dots K$ and $G \dots K'$ to slopes $S \dots X$ and $S \dots Y$ respectively. $G \dots K$ is the minimum distance reached when on the course and speed represented by the vector $e \dots s_1$. The relative bearing is the difference between the bearing of G from K and the course indicated by vector $e \dots s_1$.

Answer.—(a) $274\frac{1}{2}^{\circ}$. (b) 9.15 miles; 20.8 minutes. (c) 180° relative or dead astern.

NOTE.—If the slopes $S \dots X$ and $S \dots Y$ are on the same side of G, the preferable course under the required conditions is always away from the Guide. If vector $e \dots s_2$ is used, then the minimum range would be $G \dots K'$ or 2.2 miles, with the cruiser bearing dead ahead of the submarine.

G must be located between the slopes $S \dots X$ and $S \dots Y$, for the submarine to make contact with the cruiser.

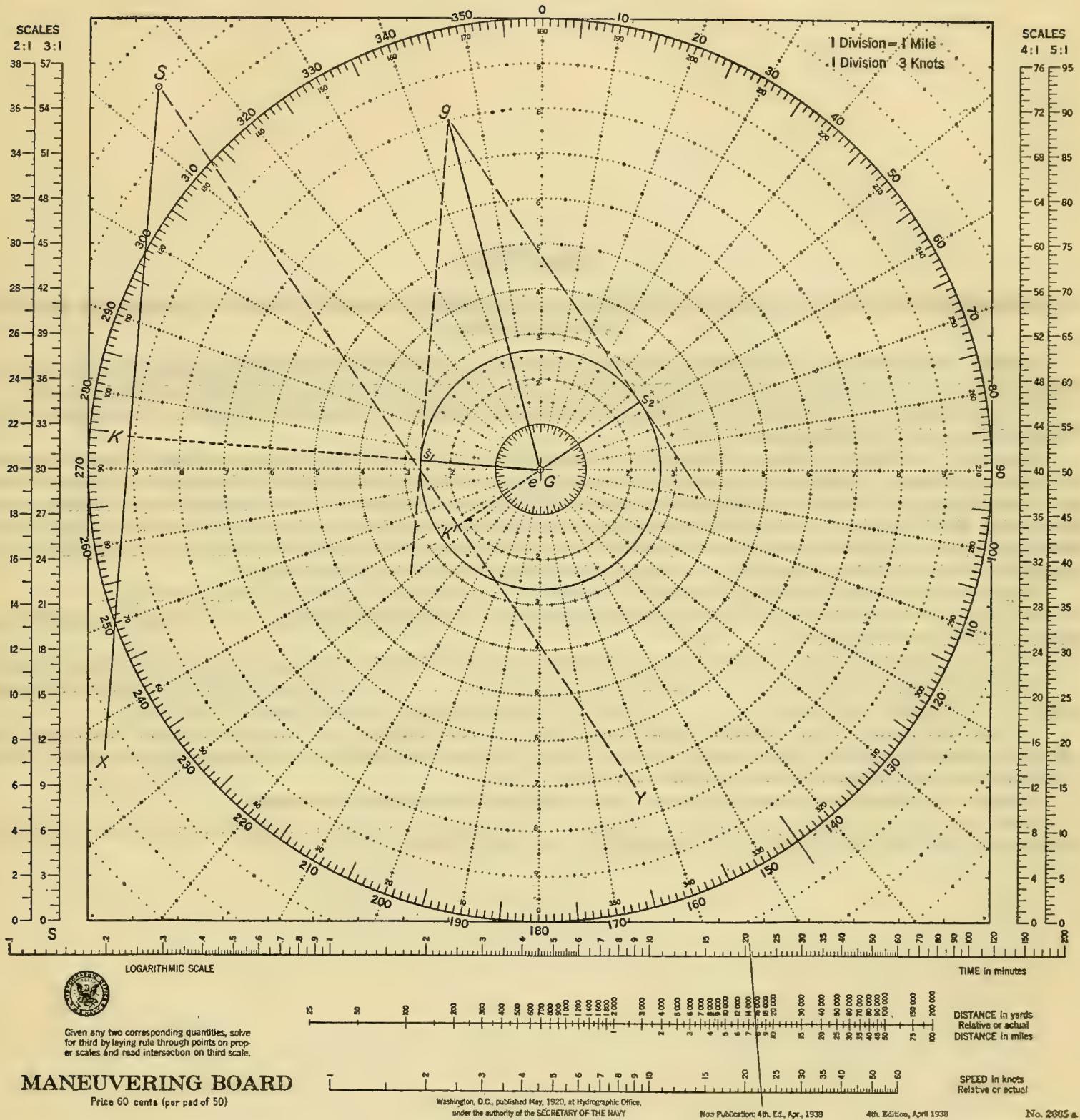


FIGURE 10.

Case VIII

TO DETERMINE COURSE AND SPEED OF GUIDE FROM TWO SETS OF SIMULTANEOUSLY OBSERVED RANGES AND BEARINGS

GIVEN: OWN COURSE AND SPEED, TWO SETS OF SIMULTANEOUSLY OBSERVED RANGES AND BEARINGS OF GUIDE FROM THE MANEUVERING UNIT, AND TIME INTERVAL BETWEEN OBSERVATIONS.

TO DETERMINE: COURSE AND SPEED OF GUIDE, AND POSITION OF MANEUVERING UNIT RELATIVE TO GUIDE AT ANY SUBSEQUENT TIME.

Example.—A destroyer rejoining a formation at a geographical point, after being on detached duty, is now on course 026° , speed 25.0 knots. The course and speed of the formation have not been signalled, since radio silence is in effect and the use of visual signals is restricted. At 1600 the first vessel in the formation is sighted and a simultaneous range of 29,000 yards and bearing of 007° obtained. This vessel is later identified as the Formation Guide, and at 1617 another observation is obtained, giving range of 20,000 yards and bearing of 014° . The commanding officer of the destroyer decides to maintain present course and speed until 1630, at which time he would head for this proper position in the formation.

Required.—(a) Course and speed of the Guide. (b) Relative position of the destroyer at 1630. (See fig. 11.)

Procedure.—Plot the Guide at G and the relative positions of the destroyer at 1600 and 1617 at D_1 and D_2 respectively. Measure the Relative Distance $D_1 \dots D_2$.

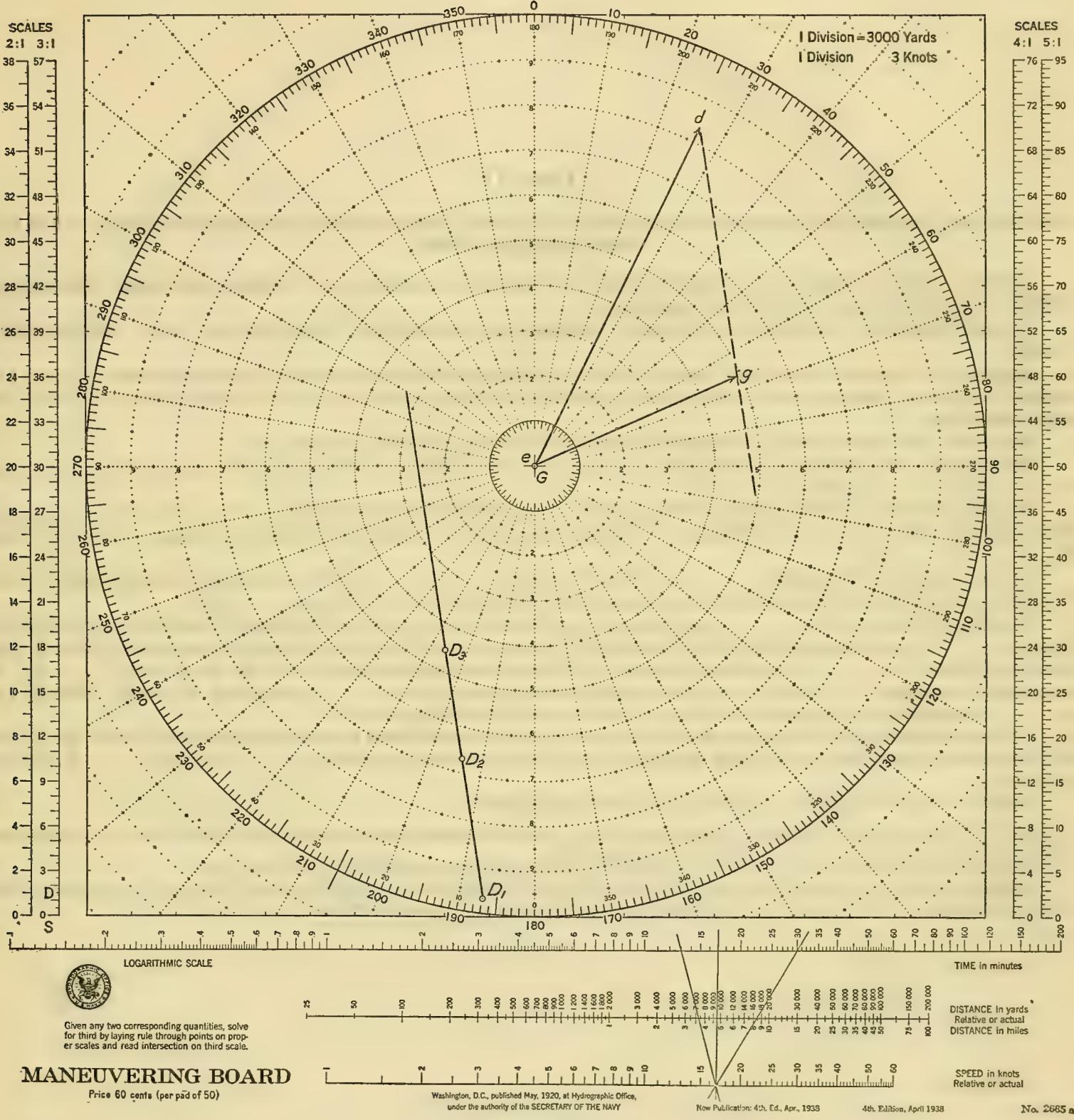
Using the Logarithmic Scale, the time between observations, and the Relative Distance, determine the Relative Speed.

Draw the destroyer's vector $e \dots d$, and transfer the slope $D_1 \dots D_2$ to d . Using the Relative Speed found above measure it on this slope, locating g . $e \dots g$ is the vector representing the course and speed of the Guide.

To locate the position of the destroyer relative to the Guide at 1630, continue the Relative Movement Line beyond D_2 and use the Logarithmic Scale. The estimated position D_3 may be plotted from either D_1 or D_2 , depending upon whether 30 or 13 minutes are used for length of time on Relative Movement Line.

Answer.—(a) Course 066° , speed 14.7 knots. (b) 13,600 yards, bearing 206° from Guide.

NOTE.—The vessel which is taking the bearings and ranges may be designated as the Guide in some problems, in which case it should be located first, and the other vessel plotted from the Guide's position. The solution is similar.



Case IX

TO FIND THE COURSE AND RELATIVE POSITION OF A VESSEL FROM THREE OR MORE BEARINGS, KNOWING THE VESSEL'S SPEED

GIVEN: OWN COURSE AND SPEED, SPEED OF SECOND VESSEL, AND THREE OR MORE BEARINGS WITH TIME INTERVAL BETWEEN BEARINGS.

TO DETERMINE: COURSE OF SECOND VESSEL AND RELATIVE POSITION AT ANY SUBSEQUENT TIME.

Example.—A ship *A* on course 350° , speed 14.0 knots, is taking radio bearings of ship *B*, which latter vessel is known to be making 10.0 knots. Bearings taken at 0813, 0858, and 0928, after being converted to true bearings, yield $017\frac{1}{2}^\circ$, 035° , and 050° , respectively.

Required.—(a) Course of *B*. (b) Range and bearing of *B* from *A* at 1000. (See fig. 12.)

Procedure.—Plot position of ranging ship *A* at any convenient point, and from this point lay out observed bearings $A \dots p_1$, $A \dots p_2$, and $A \dots p_3$.

At any point lay out a slope across the bearing lines so inclined that intercepts between bearings are proportional to the time intervals between bearings. Methods of accomplishing this are shown on the following page. Letter this slope $P_1 \dots P_2 \dots P_3$.

Lay out ranging ship's vector $e \dots a$. Transfer slope $P_1 \dots P_2 \dots P_3$ to a , cutting 10.0-knot speed circle at b_1 and b_2 . *B* therefore has two possible courses represented by vectors $e \dots b_1$ and $e \dots b_2$.

Assuming *B*'s vector to be $e \dots b_1$, by using the time between the first and third bearings (75 minutes) and the Relative Speed $a \dots b_1$ on the Logarithmic Scale, the Relative Distance *B* would run in this period of time is easily found. This Relative Distance $B_1 \dots B_3$ is plotted between the 0813 and the 0928 bearings parallel to the slope $P_1 \dots P_3$. B_3 represents the position of *B* relative to *A* at 0928 under the assumption that $e \dots b_1$ represents *B*'s vector. Similarly, if $e \dots b_2$ is assumed to be proper vector for *B*, its relative position at 0928 plots at B'_3 .

To find the Relative Positions possible for *B* at 1000, the respective Relative Speeds for the total time of 107 minutes on the Logarithmic Scale, give the Relative Distances run on courses shown by vectors $e \dots b_1$ and $e \dots b_2$ locating B_4 and B'_4 .

Answer.—(a) 108° or $001\frac{1}{2}^\circ$. (b) 39.4 or 8.8 miles, bearing 067° .

NOTE.—When the speed of the second vessel is less than the speed of the ranging vessel, two courses will be obtained for the second vessel except when the transferred slope is tangent to the known speed circle.

In case the bearing does not change, it will not be possible to determine the Relative Position of *B* by this method.

All solutions obtained by bearings alone should be used with caution as a wide final error may result from a comparative small error in taking one or more bearings. This is borne out by the Radian Rule that "An error of 1° is an error of 1 mile at a distance of 60 miles."

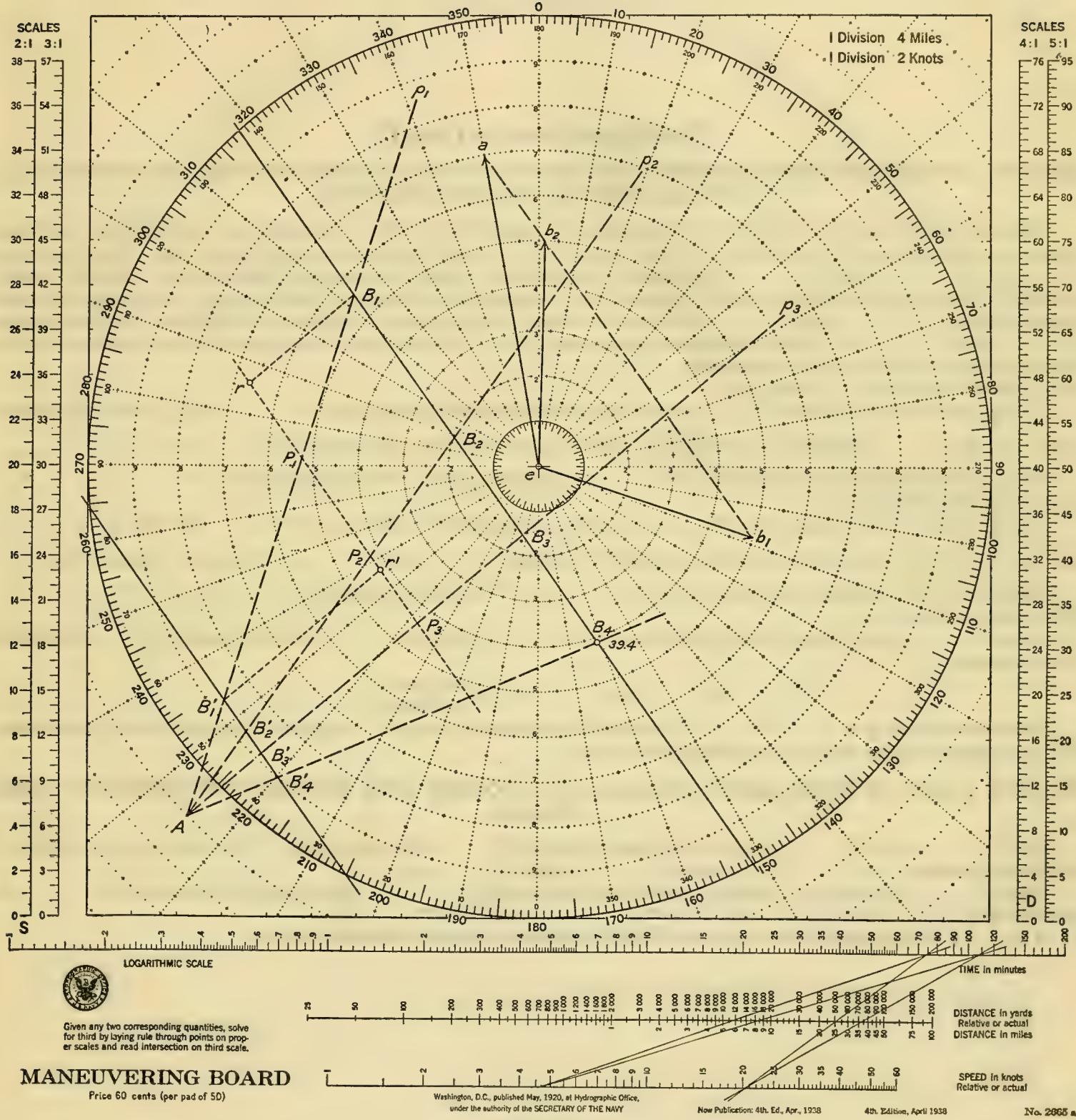


FIGURE 12.

Techniques Used in Case IX

TO DRAW A LINE ACROSS THREE BEARING LINES SO INCLINED THAT INTERCEPTS WILL BE PROPORTIONAL TO TIME INTERVALS BETWEEN BEARINGS

Although a trial and error solution of this problem is possible by the use of parallel rulers and dividers, the three methods shown below have been found to be more expeditious. In each method the same data is used. Three bearings, $O \dots a$, $O \dots b$, and $O \dots c$, are taken by an observing vessel O of a second vessel, with an interval of time t_1 between the first and second bearings and an interval of time t_2 between the second and third bearings. The angular difference between bearing $O \dots a$ and $O \dots b$ is given by angle X , while angle Y indicates the difference between bearings $O \dots b$ and $O \dots c$. These methods are shown sketched on a Maneuvering Board because of the convenience of the attached scales, but this is not a requirement.

First Method (Sketch a). (See fig. 13.)

At any point P on bearing line $O \dots a$ erect a perpendicular cutting bearing line $O \dots b$ at B . Along this perpendicular lay off $B \dots Q$ so that $\frac{P \dots B}{B \dots Q} = \frac{t_1}{t_2}$. Erect a perpendicular at Q , cutting $O \dots c$ at C .

Connect B and C and extend this line to $O \dots a$, intersecting at A . The required slope is $A \dots B \dots C$.

Second Method (Sketch b).

At any point P on bearing line $O \dots b$ erect a perpendicular and lay out $P \dots Q$ and $P \dots R$ so that $\frac{P \dots Q}{P \dots R} = \frac{t_1}{t_2}$. Erect perpendiculars at Q and R , cutting $O \dots a$ and $O \dots c$ at A and C respectively.

Connect A and C , cutting the bearing line $O \dots b$ at B . The required slope is $A \dots B \dots C$.

Third Method (Sketch c).

Along $O \dots a$ lay out any convenient distance $O \dots A$. Along $O \dots c$ lay out length $O \dots C$ obtained by the formula:

$$O \dots C = O \dots A \times \frac{t_2}{t_1} \times \frac{\sin \text{angle } X}{\sin \text{angle } Y}$$

The required slope is $A \dots B \dots C$, obtained by connecting A and C . In case both angles X and Y are less than 16° , the numerical values may be used instead of the sines.

TO DRAW A LINE OF GIVEN LENGTH PARALLEL TO ANOTHER LINE, BETWEEN TWO DIVERGENT LINES (Sketch d)

Let $O \dots a_1$ and $O \dots a_2$ be two divergent lines and $P \dots Q$ a given slope. It is required to draw a line of given length, parallel to $P \dots Q$, between $O \dots a_1$ and $O \dots a_2$.

Along $P \dots Q$, extended if necessary, lay out the given length $P \dots R$. From R draw a line parallel to $O \dots a_1$, intersecting $O \dots a_2$ at C . Through C draw $A \dots C$ parallel to $P \dots Q$.

$A \dots C$ is the required line, the proof of which is apparent from the parallelogram law.

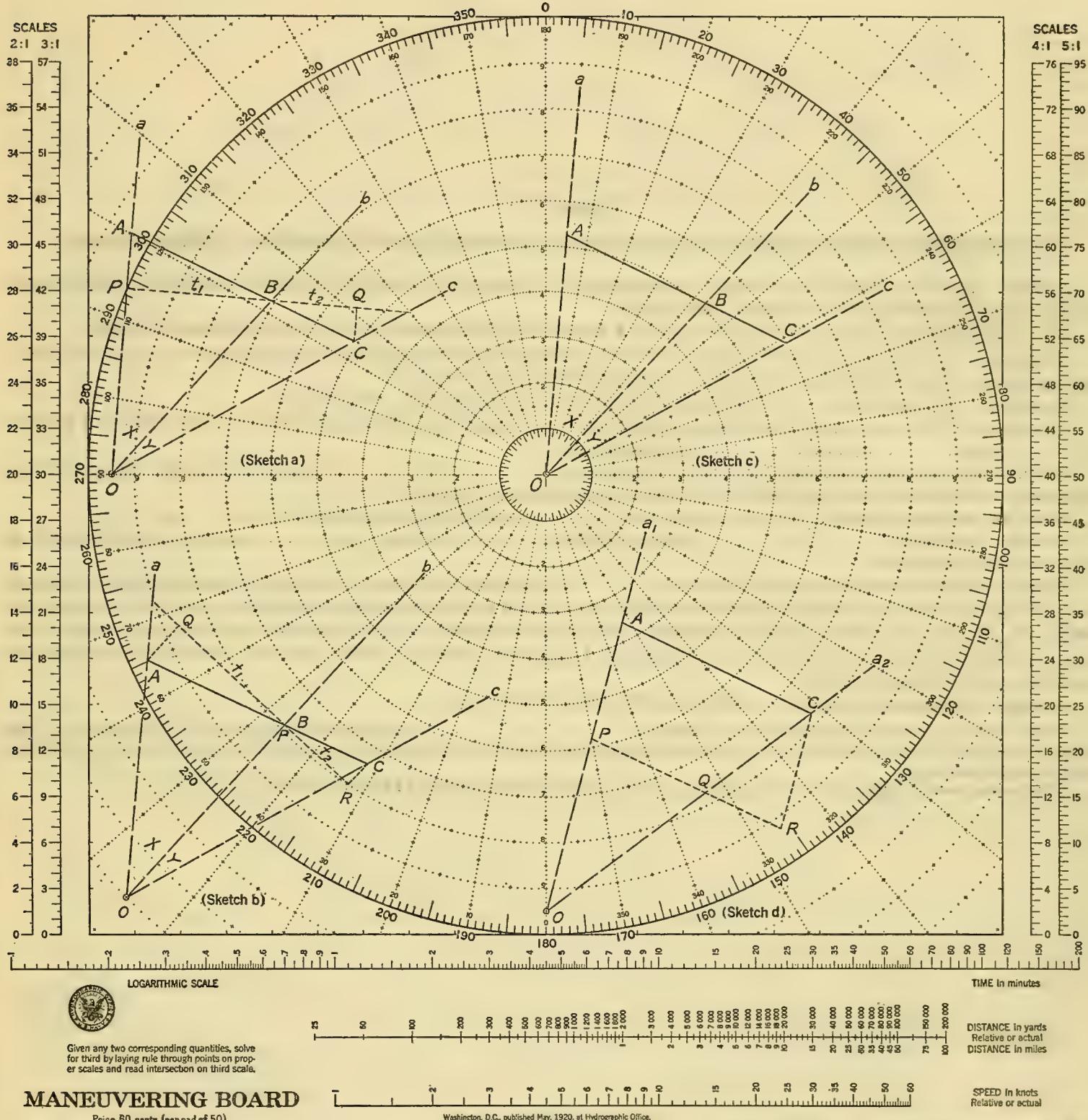


FIGURE 13.

Case X

TO DETERMINE SPEED OF GUIDE FROM THREE OR MORE BEARINGS, KNOWING GUIDE'S COURSE

GIVEN: OWN COURSE AND SPEED, COURSE OF GUIDE AND THREE OR MORE BEARINGS OF GUIDE WITH TIME INTERVALS BETWEEN BEARINGS.

TO DETERMINE: SPEED OF GUIDE AND RELATIVE POSITION AT ANY SUBSEQUENT TIME.

Example.—Ship A, on course 140° at speed 12.0 knots, is taking bearings of ship M, which is known to be on course 180° . Bearings taken at 1130, 1230, and 1310 are 090° , 117° , and $149\frac{1}{2}^\circ$ respectively.

Required.—(a) Speed of M. (b) Estimated relative position of M at 1350. (See Fig. 14.)

Procedure.—Plot the ranging vessel at any convenient point A and lay out bearings of M at 1130, 1230, and 1310 as $A \dots p_1$, $A \dots p_2$, and $A \dots p_3$, respectively.

By any of the methods described for case IX, draw slope $P_1 \dots P_2 \dots P_3$ across the bearing lines at such an angle that the intercepts are proportional to the time intervals between bearings.

From any convenient point e, lay out A's vector $e \dots a$, and M's known course line $e \dots m'$.

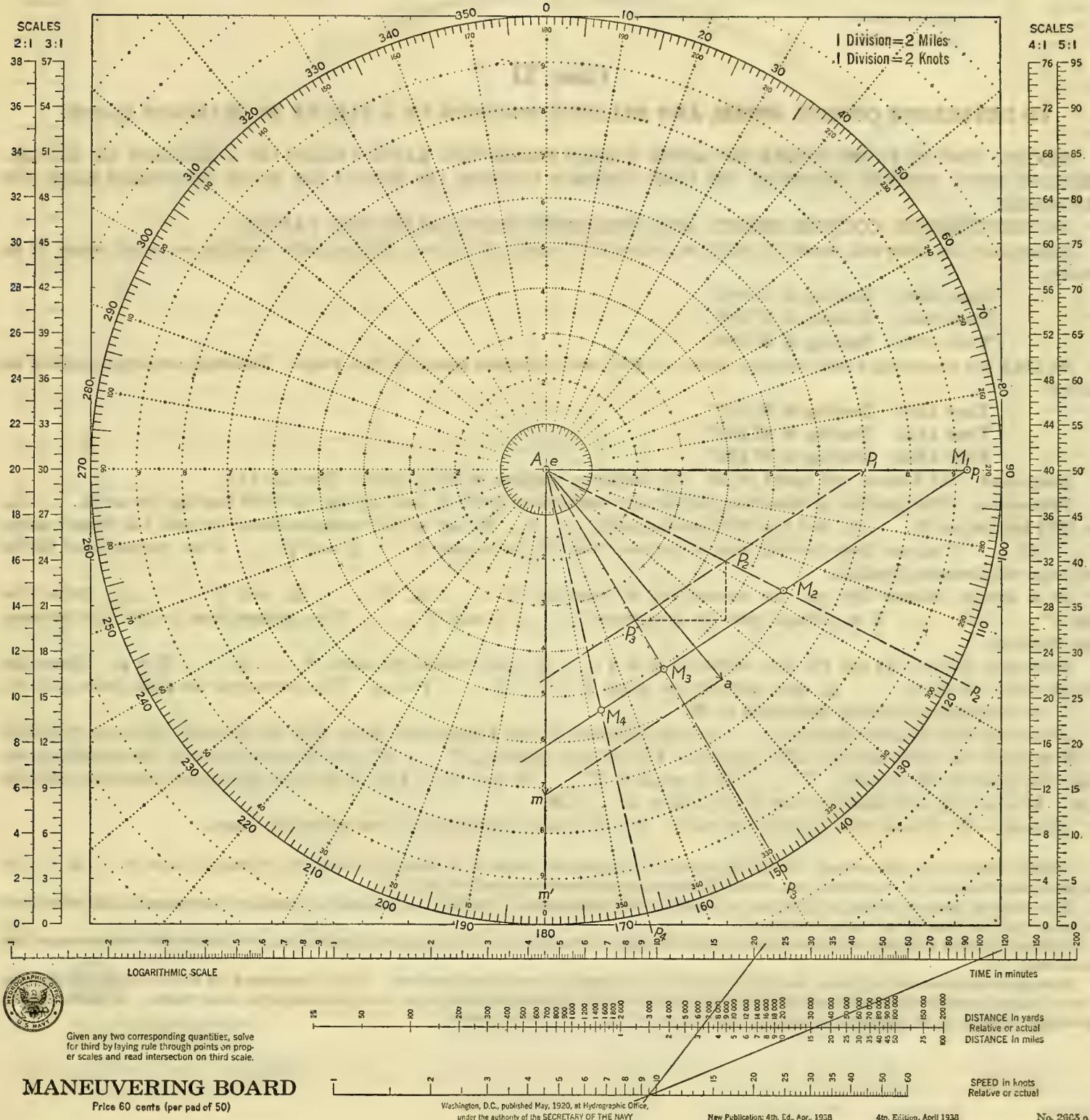
Transfer the slope $P_1 \dots P_2 \dots P_3$ to a, cutting $e \dots m'$ at m. $e \dots m$ is the vector representing the course and speed of M.

Using the Relative Speed indicated by vector $a \dots m$, the Relative Distance run by M during the time the bearings were taken is found by means of the Logarithmic Scale. By trial and error or the method indicated in case IX, this is plotted, locating M_1 , M_2 , and M_3 . These positions are those occupied by M relative to A at 1130, 1230, and 1310, respectively. The relative position of M at 1350 is also found through the Logarithmic Scale as a $2^h 20^m$ run from 1130 or a 40-minute run from 1310, and is designated as M_4 .

Answer.—(a) Speed 14.5 knots. (b) Bearing 167° , distant 10.7 miles.

NOTE.—This case becomes indeterminate if the slope $P_1 \dots P_2 \dots P_3$ is parallel to the vector of the ranging ship. This can occur only if M is on the same course as A or the reverse of this course. In this event, since the course of M is known, A should steer another course for taking bearings.

If the bearing does not change, it will not be possible to determine the relative position of M by this method.



Case XI

TO DETERMINE COURSE, SPEED, AND RELATIVE POSITION OF A TARGET BY BEARINGS ALONE

GIVEN: TWO SETS OF THREE OR MORE TIMED BEARINGS, EACH TAKEN ON A TARGET BY AN OBSERVING UNIT WHICH CHANGES ITS OWN KNOWN COURSE OR SPEED OR BOTH BETWEEN SETS OF BEARINGS.

TO DETERMINE: COURSE, SPEED, AND RELATIVE POSITION OF THE TARGET.

Example.—An observing vessel G , while on course 110° , speed 15.0 knots, obtains radio bearings on target vessel M as follows:

Time 0800. Bearing of $M 000^\circ$.

Time 0900. Bearing of $M 349^\circ$.

Time 1000. Bearing of $M 336^\circ$.

At 1015 the observing vessel changes course to 045° and increases speed to 20.0 knots. Bearings are next received as follows:

Time 1030. Bearing of $M 327^\circ$.

Time 1130. Bearing of $M 302^\circ$.

Time 1230. Bearing of $M 273^\circ$.

Required.—(a) Course and speed of M . (b) Relative position of M at 1230. (See fig. 15.)

Procedure.—Plot position of observing ship at any convenient point G , and lay out the 0800 bearing line as $G \dots b_1$, the 0900 bearing as $G \dots b_2$, and the 1000 bearing as $G \dots b_3$. By any of the methods shown for case IX, draw a slope $P \dots Q \dots R$ across these bearing lines so inclined that the intercepts $P \dots Q$ and $Q \dots R$ are proportional to the time intervals between bearings.

In a similar manner lay out the second set of bearings, $G \dots b_4$, $G \dots b_5$, and $G \dots b_6$. Draw the slope $T \dots U \dots V$ so inclined that the intercepts $T \dots U$ and $U \dots V$ are proportional to the time between these bearings.

From any point e , lay out the first vector of G as $e \dots g_1$, and transfer the slope $P \dots Q \dots R$ to g_1 . Draw the second vector of G , $e \dots g_2$, and transfer the slope $T \dots U \dots V$ to g_2 . This intercepts the slope from g_1 at m . $e \dots m$ represents the course and speed of M .

Determine the Relative Speed $g_2 \dots m$ and by means of the Logarithmic Scale find the Relative Distance travelled by M between the 1030 and the 1230 bearings. Lay off this distance, $T \dots W$, and by completion of the parallelogram locate the position M at 1230. $T' \dots M$ is equal to $T \dots W$ and is the Line of Relative Movement between 1030 and 1230. M 's bearing and distance from G at 1230 is $G \dots M$.

Answer.—(a) Course 164° , speed 12.6 knots. (b) 57.5 miles bearing 273° from observing vessel.

NOTE.—Solution by this method will not be obtained if the second vector of G should lie along the transferred slope $P \dots Q \dots R$. For this reason this slope is transferred to g_1 before G changes either course or speed or both.

If the bearing does not change in either set, a solution is still obtainable. The slope in this case is a constant bearing and is laid off in both directions from the head of the vector concerned.

In case G makes a change of course when the last bearing of the first set is obtained, this bearing may be used as the first bearing of the second set.

When bearings alone are available, the results should be considered as approximations only. This is occasioned by the fact that a small error in one or more bearings will change the inclination of the slopes to be transferred, and this in turn will change the final results. If sufficient time is available, a third set of bearings will act to check the course and speed of the target.

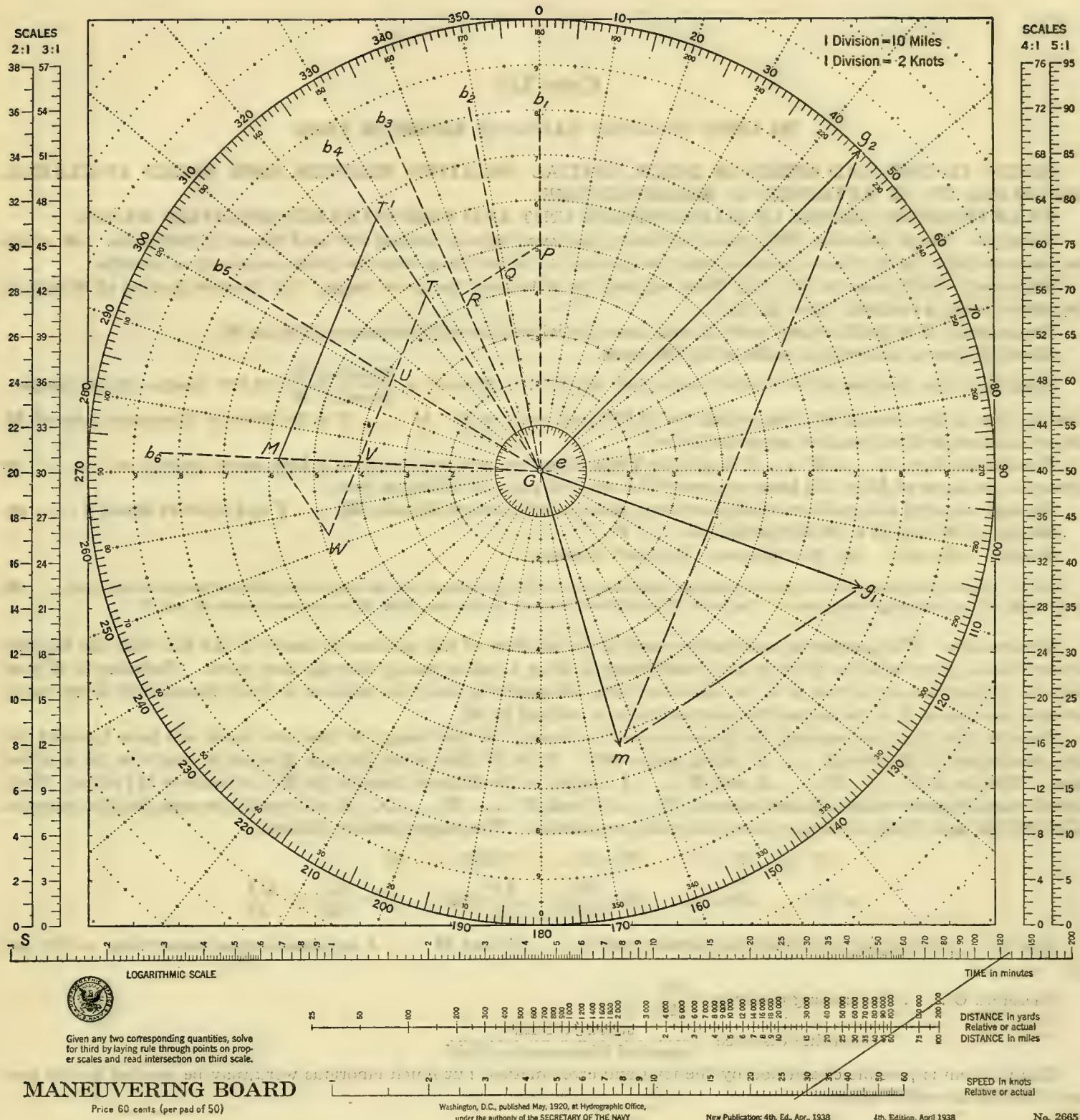


FIGURE 15.

Case XII

TO OPEN TO GIVEN RANGE IN MINIMUM TIME

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL RELATIVE POSITION, OWN SPEED AVAILABLE, AND RANGE TO BE ATTAINED IN MINIMUM TIME.

TO DETERMINE: COURSE OF MANEUVERING UNIT AND TIME TO REACH SPECIFIED RANGE.

Example.—Guide on course 300° , speed 14.0 knots, has vessel M now bearing 000° and distant 15,000 yards. M , which has power available for 21.0 knots, receives orders to open out to 30,000 yards from the Guide as quickly as possible.

Required.—(a) Course for M . (b) Time required for M to reach the given range. (c) Relative bearing of M from G when given range is reached. (See fig. 16.)

Procedure.—Plot Guide at any point G , and locate initial position of Maneuvering Unit at M .

About G draw a circle with a radius of 30,000 yards.

From G , in the direction of the Guide's course, lay out $G \dots X$ equal to $\frac{\text{Speed of } G}{\text{Speed of } M} \times 30,000$ yards = 20,000 yards.

From X , draw a line through M , intersecting the 30,000-yard circle at Y . $M \dots Y$ is the Relative Movement Line of M . The explanation for the formula is appended below.

Transfer the slope $X \dots M \dots Y$ to g in the Vector Diagram, intersecting the 21.0 knot speed circle at m . $e \dots m$ represents the course of M at 21.0 knots to open to the required range in minimum time.

The time required is found from the Logarithmic Scale, using Relative Distance $M \dots Y$ and Relative Speed $g \dots m$.

Bearing of M from G at 30,000 yards is $G \dots Y$.

Answer.—(a) 046° . (b) 23.7 minutes. (c) 106° relative.

NOTE.—Should the speed of G be greater than the speed available to M , the point X will lie outside the specified range circle and $X \dots M$, as extended, will cut the range circle twice. The point Y should be so chosen, in this case, that M lies between points X and Y .

Explanation.—The geometric construction used in the solution of this problem is based on the fact that the shortest distance from a point within a circle to its circumference is along a radius passing through that point. Therefore, if M is to reach the 30,000-yard circle from G in the minimum time, M must run along the radius of a circle whose center is the *navigational* position of G at the instant the circumference is reached by M .

Consider that the time required for this evolution is t hours. In that length of time the Guide will have traveled $14t$ miles, plotted as the distance $G \dots G'$, along course 300° . From G' draw a line through M equal in length to the specified range. This line is $G' \dots M \dots A$, and $M \dots A$ is equal to the distance traveled by M in t hours or $21t$ miles. Complete the parallelogram whose adjacent sides are $G \dots G'$ and $G' \dots M \dots A$. It will readily be seen that the triangles XGY , $XG'M$, and YAM are similar since their sides are parallel. The proportionality follows:

$$(G \dots X):(A \dots Y)=(G \dots Y):(M \dots A)$$

$$\text{or } (G \dots X)=(A \dots Y) \times \frac{(G \dots Y)}{(M \dots A)}=(G \dots Y) \times \frac{(G \dots G')}{(M \dots A)}$$

Now $G \dots Y$ is equal to the limiting distance and $G \dots G'$ and $M \dots A$ are drawn equal respectively to $14t$ and $21t$.

$$\begin{aligned} \text{Therefore } G \dots X &= \text{limiting distance} \times \frac{14t}{21t} \\ &= \text{limiting distance} \times \frac{\text{speed of Guide}}{\text{speed of Maneuvering Unit}} \end{aligned}$$

This problem is possible of solution by the trial and error method but much laborious work may be avoided by the geometric method described above.

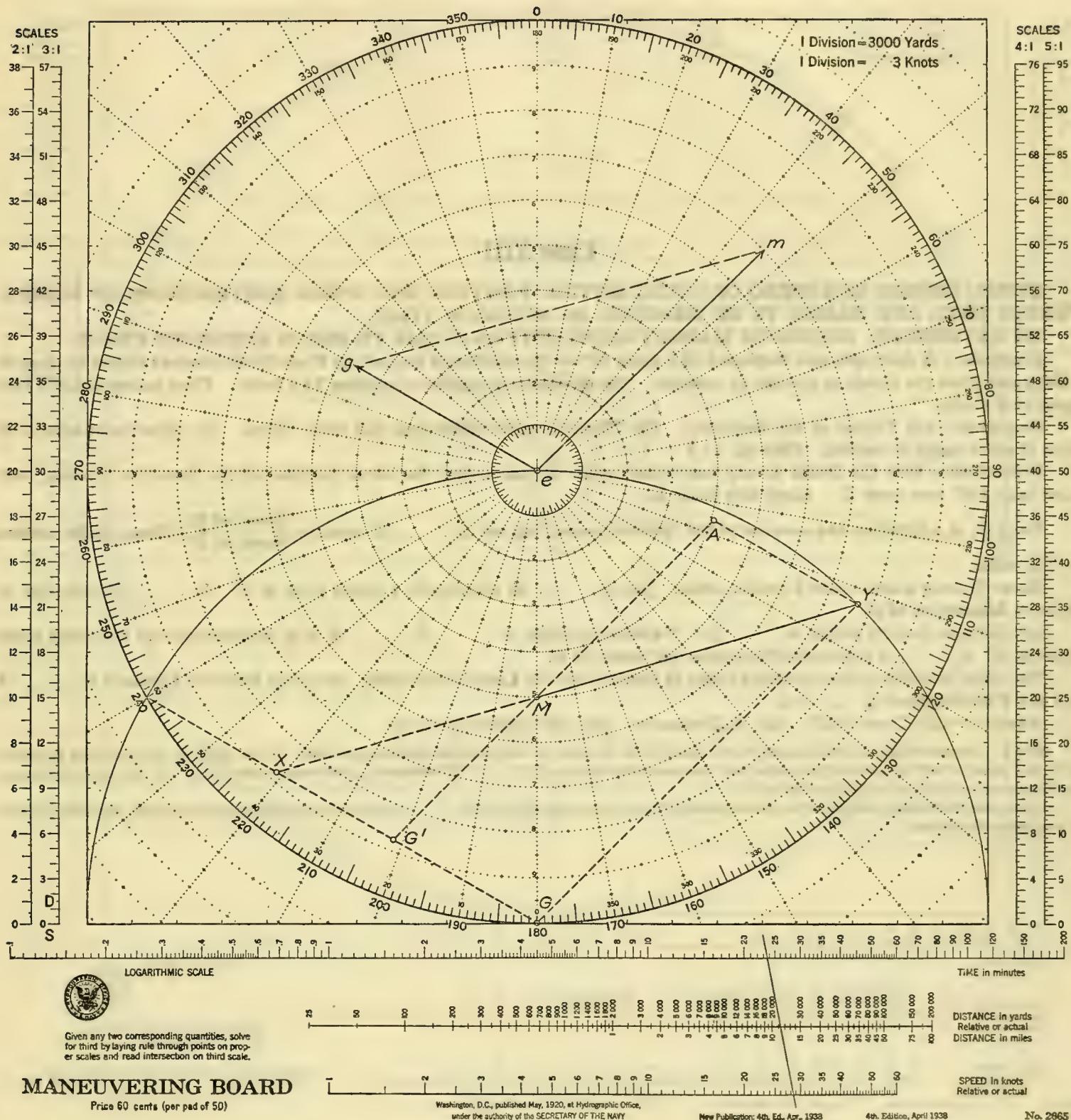


FIGURE 16.

Case XIII

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL POSITION AND SPEED RESTRICTIONS OF MANEUVERING UNIT, AND RANGE TO BE REACHED IN MINIMUM TIME.

TO DETERMINE: COURSE OF MANEUVERING UNIT AND TIME TO REACH SPECIFIED RANGE.

Example.—A destroyer now stationed 10.0 miles 80° on the starboard bow of the Fleet Guide receives orders to close to 4,000 yards from the Guide as quickly as possible. The destroyer is capable of making 24.0 knots. Fleet course is 020° , fleet speed 15.0 knots.

Required.—(a) Course of the destroyer. (b) Time to reach 4,000-yard (2.0 mile) range. (c) Relative bearing of D from G when range is reached. (See fig. 17.)

Procedure.—Plot the Guide at any convenient point G , and locate the initial position of the destroyer, bearing 080° relative or 100° true from G . Mark this point D .

From G , in a direction the reverse of the Guide's course, lay out $G \dots X$ equal to $\frac{\text{Speed of } G}{\text{Speed of } D} \times \text{Range to be reached}$ or 1.25 miles.

About G draw a circle with 2.0 mile radius. Join $D \dots X$, cutting the 2.0 mile circle at Y . $D \dots Y$ is the line of Relative Movement of D .

Lay out the Guide's vector $e \dots g$. Transfer the slope $D \dots Y \dots X$ to g , intersecting the 24.0-knot speed circle at d . $e \dots d$ represents the course and speed of D .

The time to arrive at the specified range is found from the Logarithmic Scale, using the Relative Distance $D \dots Y$ and the Relative Speed $g \dots d$.

Answer.—(a) Course 310° . (b) 21.3 minutes. (c) 110° relative from G .

NOTE.—In case the speed of D is equal to the speed of G , the point X will lie in the given range circle. If the speed of D is less than the speed of G , the point X will lie outside the circle, and the possibility exists of $D \dots X$ intersecting the range circle twice. In this event, the intersection nearer to D is the one to be used as Y .

In the case the initial position of the Maneuvering Unit is such that the line $D \dots X$ does not intercept the range circle, then the problem is incapable of solution.

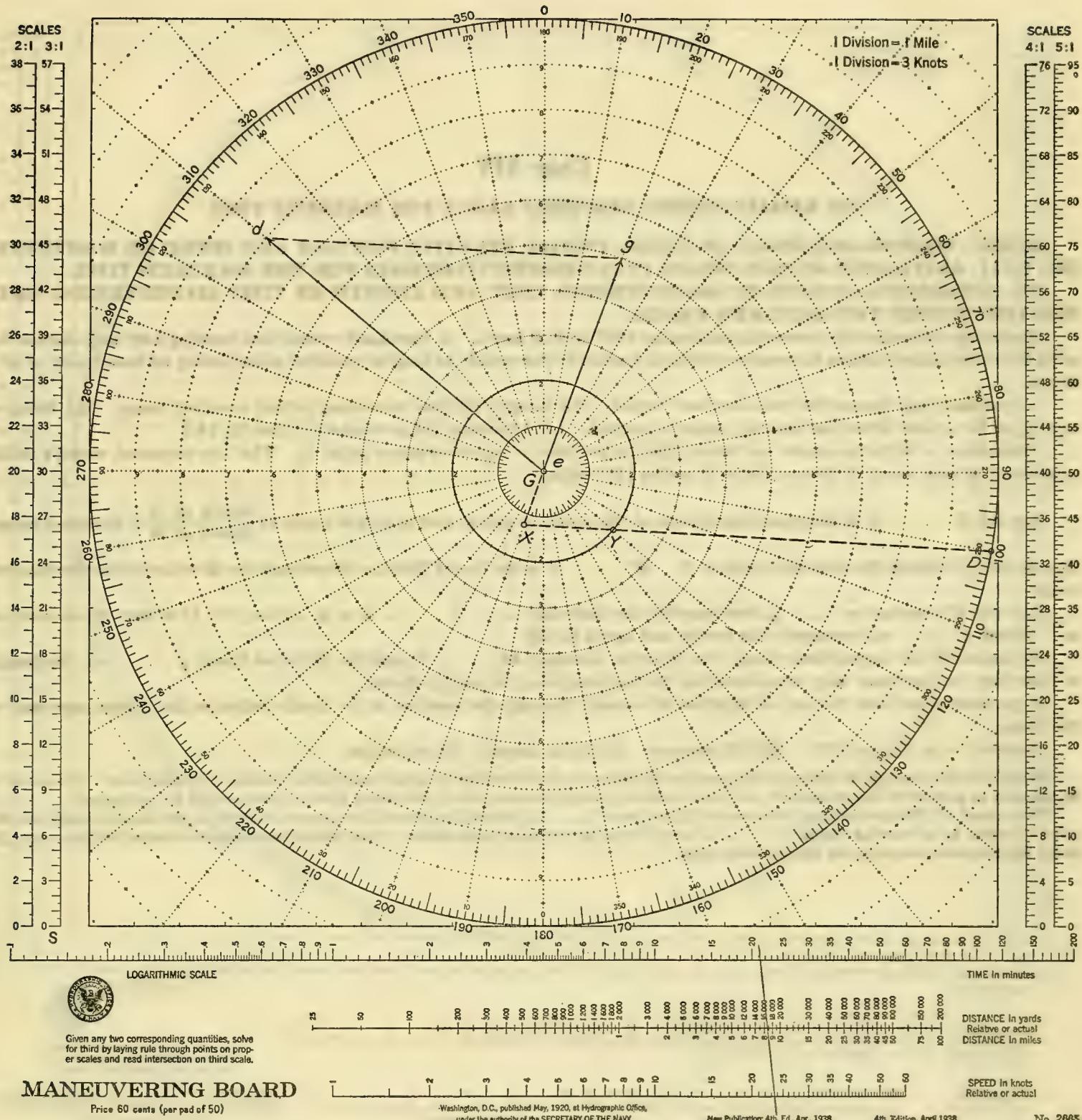


FIGURE 17.

Case XIV

TO REMAIN WITHIN SPECIFIED RANGE FOR MAXIMUM TIME

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL RELATIVE POSITION AND SPEED OF MANEUVERING UNIT, AND RANGE WITHIN WHICH IT IS DESIRED TO REMAIN FOR THE MAXIMUM TIME.

TO DETERMINE: COURSE OF MANEUVERING UNIT AND LENGTH OF TIME MANEUVERING UNIT REMAINS WITHIN THE REQUIRED RANGE.

Example.—A cruiser is proceeding on course 190° at 21.0 knots. A merchant vessel, now bearing 280° and distant 5.0 miles from the cruiser desires to remain within 10.0 miles of the cruiser as long as possible; maintaining its best speed of 12.0 knots.

Required.—(a) Course for the merchant vessel. (b) Length of time remaining within specified range. (c) Relative bearing of the cruiser when the limiting range is reached. (d) Minimum range reached. (See fig. 18.)

Procedure.—With the cruiser as Guide, plot its position at any convenient point G. Plot the merchant vessel's initial position at M and about G draw a circle of radius 10.0 miles.

Lay off G X in direction the reverse of the cruiser's course and in length equal to $\frac{\text{Speed of } G}{\text{Speed of } M} \times \text{Specified range}$.

Join M and X, cutting the 10.0 mile circle at Y. M Y is the line of Relative Movement for M while within the required range.

Lay out G's vector, e g, and transfer the slope M Y X to g, cutting the 12.0 knot speed circle at m. Vector e m represents the course and speed for M.

By means of the Logarithmic Scale, the Relative Distance M Y, and the Relative Speed g m, the length of time that the merchant ship remains within the required range is readily found.

A perpendicular from the line of Relative Movement through the position of the Guide indicates the closest approach to the Guide.

Answer.—(a) Course 177° . (b) 63 minutes. (c) Dead ahead. (d) 4.8 miles.

NOTE.—If the speed of M is equal to the speed of G, the point X will fall on the given range circle, locating Y at this point. The course of M will then be parallel to the course of G, and since both are making the same speed, the initial relative positions will be maintained.

If the speed of M is greater than the speed of G, the line G X should be plotted in the same direction as the Guide's course and the point Y located by extending the line G X until it cuts the required range circle. When the required range is reached, in this event, the Guide will bear dead astern of the Maneuvering Unit.

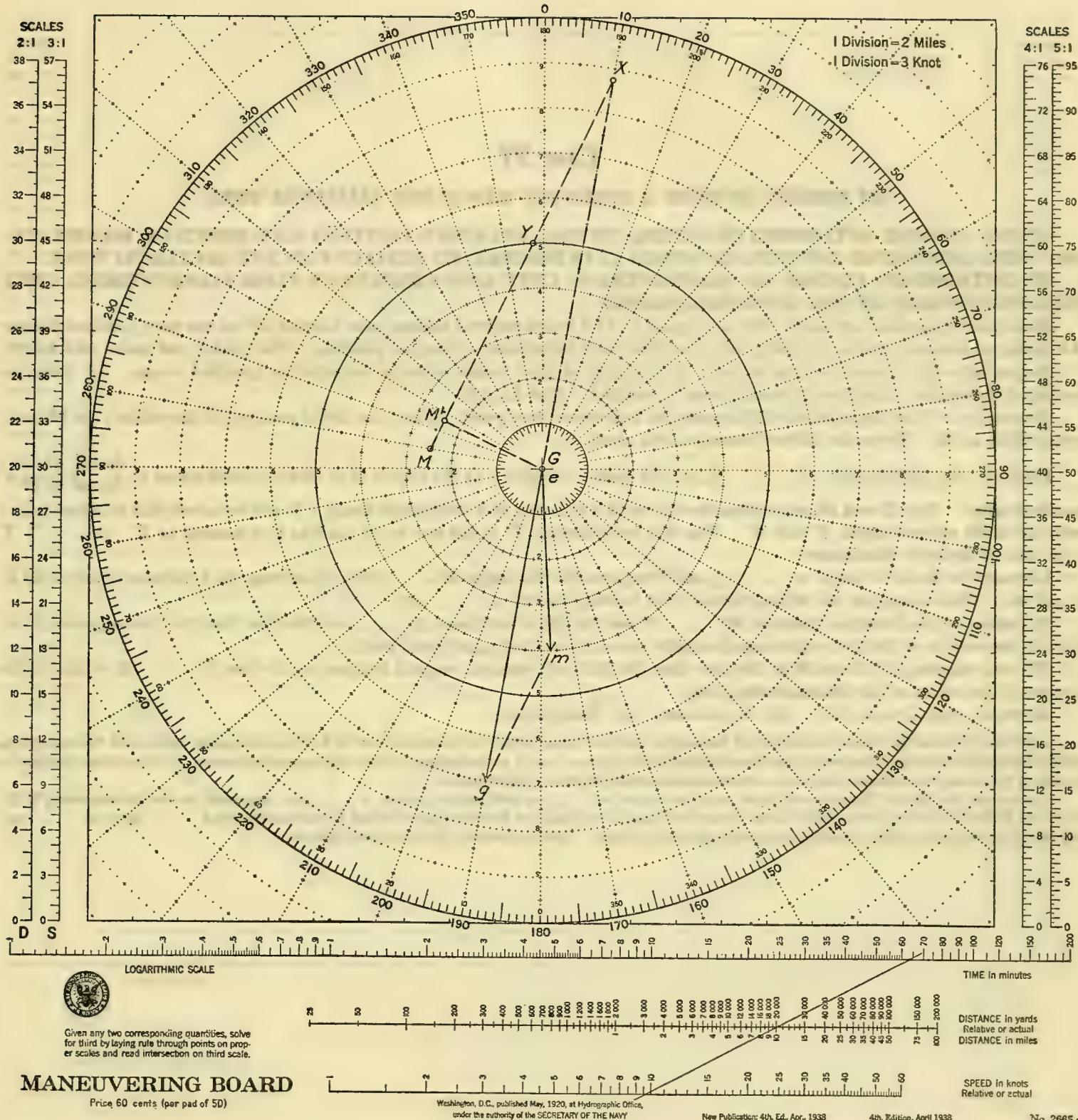


FIGURE 18.

Case XV

TO REMAIN OUTSIDE A SPECIFIED RANGE FOR MAXIMUM TIME

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL RELATIVE POSITION AND SPEED OF MANEUVERING UNIT, AND RANGE OUTSIDE OF WHICH IT IS DESIRED TO REMAIN FOR THE MAXIMUM TIME.

TO DETERMINE: COURSE OF MANEUVERING UNIT AND RESULTANT TIME MANEUVERING UNIT REMAINS OUTSIDE OF THE SPECIFIED RANGE

Example.—A cruiser on course 210° at a speed of 18.0 knots orders a tanker, now located 10° on the port bow and distant 14.0 miles, to remain outside of a range of 10.0 miles from the cruiser as long as possible. The tanker can make 10.0 knots.

Required.—(a) Course for the tanker. (b) Length of time tanker remains outside the specified range. (c) Relative bearing of the cruiser when the 10.0-mile range is reached. (See fig. 19.)

Procedure.—Plot the position of the cruiser at any convenient point, G , and the initial position of the tanker, the Maneuvering Unit, at M . Draw the 10.0-mile-range circle about G .

From G , lay off the line $G \dots X$, in the same direction as the course of G and in length equal to $\frac{\text{Speed of } G}{\text{Speed of } M} \times$

Specified range. Join X and M , extending the line until it cuts the 10.0-mile-range circle. It will be noted that this line intersects the circle at two points, Y and Y' . The first intersection, Y , is the one to be used as it is nearest to X . $M \dots Y$ is the line of Relative Movement.

Lay out the Guide's vector $e \dots g$, and then transfer the slope $M \dots Y$ to g , cutting the 10.0-knot-speed circle at m and m' . The course for the Maneuvering Unit is indicated by $e \dots m$.

By means of the Relative Distance $M \dots Y$ and the Relative Speed $g \dots m$, the time that the Maneuvering Unit will remain outside of the 10.0-mile range is readily found from the Logarithmic Scale.

The true bearing of G from M at the time that the 10.0-mile range is reached is given by the line $Y \dots G$, which is the reverse of the course of the Maneuvering Unit.

Answer.—(a) Course 175° . (b) 35 minutes. (c) Dead astern.

NOTE.—Unless the initial position of M lies within the area bounded by the tangents from X to the given range circle, and the arc of this circle between the points of tangency, the Maneuvering Unit need not worry about coming within the prescribed range. If the initial position is outside of this area, the Maneuvering Unit can remain outside of the range indefinitely.

A glance at the Vector Diagram will show why the vector $e \dots m$ and not the vector $e \dots m'$ was used for the Maneuvering Unit. While the Relative Distance remains the same before the range is reached, the Relative Speed would be increased from $g \dots m$ to $g \dots m'$ and the time remaining before reaching the range thereby reduced. This is contrary to the results desired.

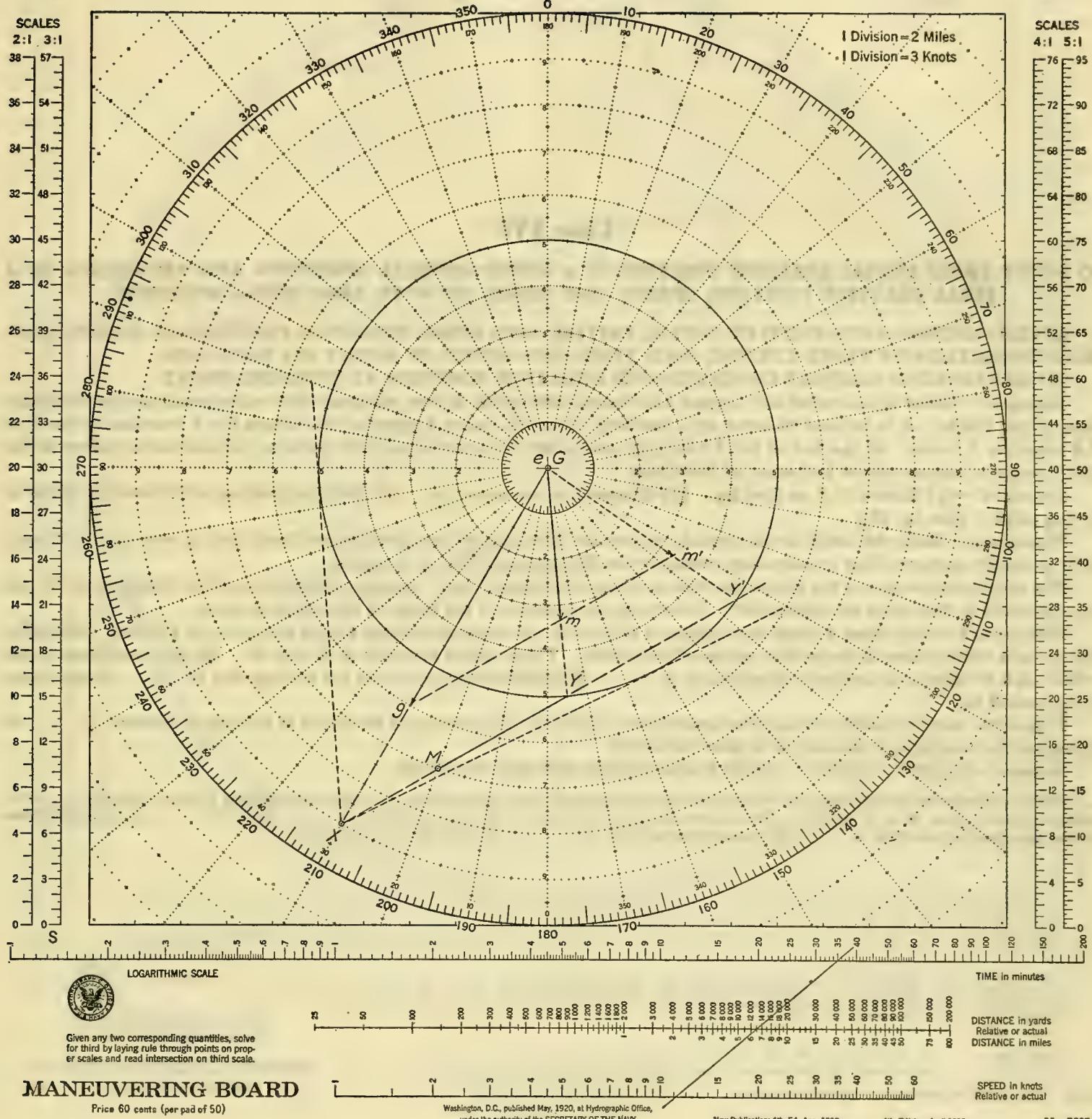


FIGURE 19.

Case XVI

TO SCOUT FROM INITIAL RELATIVE POSITION IN A GIVEN GENERAL DIRECTION AND RETURNING TO A FINAL RELATIVE POSITION, SPEEDS AND TIMES ON BOTH LEGS BEING SPECIFIED

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL AND FINAL RELATIVE POSITION OF SCOUT, GENERAL DIRECTION OF FIRST COURSE, AND TIME AND SPEED OF SCOUT ON EACH LEG.

TO DETERMINE: COURSES OF SCOUT AND RELATIVE POSITION AT TURNING POINT.

Example.—Guide is on course 320° , speed 12.0 knots, with scout A now stationed 10.0 miles broad on the starboard beam of the Guide. A is ordered to scout in a northerly direction, using a speed of 20.0 knots for 4 hours and a speed of 15.0 knots for 4 hours. At the end of the 8 hours, A must reach its new station, 10.0 miles on the starboard quarter of the Guide, ready to assume course and speed of the Guide.

Required.—(a) Course of A on first leg. (b) Course of A on second leg. (c) Relative position of A from the Guide at turning point. (See fig. 20.)

Procedure.—Since the Guide is on a steady course for 8 hours and the distances are fixed that A must travel on each leg, it is readily apparent that regular chart work, or the Navigational Plot, is indicated.

Plot the initial position of the Guide and of A at G_1 and A_1 respectively. Advance the position of the Guide to G_2 for the run of 8 hours or 96.0 miles on course 320° . Locate A_2 , the position of the scout at the end of 8 hours.

With A_1 as a center, draw a circle with radius of 80.0 miles, the distance covered by the scout on the first leg. Similarly, with A_2 as a center, draw a circle with radius of 60.0 miles. These circles intersect at K and K' . By the conditions of the problem A is to scout in a northerly direction, so $A_1 \dots K$ represents the course on the first leg and $K \dots A_2$ the course on the second leg.

To find the relative position of A at the turning point, locate G' , the position of the Guide at the end of 4 hours. $G' \dots K$ represents the bearing and distance of A from the Guide.

Answer.—(a) 359° . (b) 257° . (c) 62.5 miles bearing 037° from the Guide.

NOTE.—In case the initial course for A had been specified instead of the initial speed, this course would have been laid out from A_1 , intersecting the circle about A_2 at X and X' . The courses for the second leg, in this case, would be $X \dots A_2$ or $X' \dots A_2$, depending upon whether the speed indicated by $A_1 \dots X$ divided by 4 hours or $A_1 \dots X'$ divided by 4 hours is used.

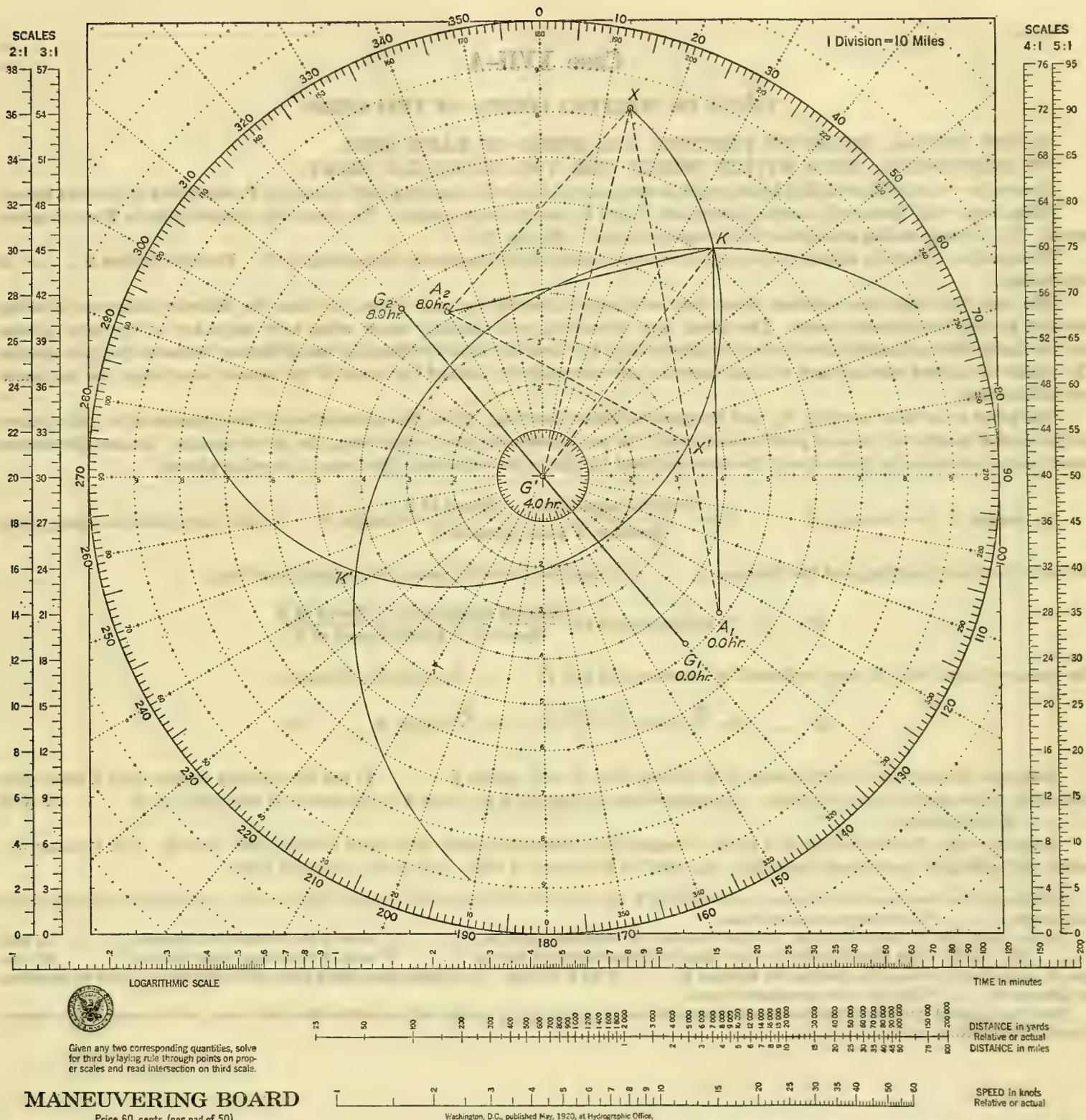


FIGURE 20

Case XVII-A

LOCUS OF MEETING POINTS OF TWO SHIPS

GIVEN: INITIAL RELATIVE POSITION AND SPEED OF EACH UNIT.

TO DETERMINE: AREA WITHIN WHICH THE TWO UNITS CAN MEET.

Example.—A ship S, with 10.0 knots speed, is now located 60.0 miles bearing 180° from ship F, which has 20.0 knots speed.

Required.—(a) Locus of meeting points for F and S, using given speeds. (b) Limiting courses open to F at its given speed. (c) Corresponding courses for S at its given speed. (See fig. 21.)

Procedure.—Plot the slower ship at any convenient place S and locate the faster ship at F. Extend the line F . . . S indefinitely.

The two ships will meet earliest when they are headed directly for each other, for then the Relative Distance is being traveled at their combined speeds. The latest time of meeting will similarly occur when both ships are on the same course and the faster ship is headed directly for the slower ship, for then the Relative Speed is the difference between the two speeds. The points of earliest meeting and of latest meeting are conveniently located by travel of the slower vessel since the distances traveled are less.

The point of earliest meeting, P_1 , and the point of latest meeting, P_2 , are the extremities of the diameter of a circle whose circumference marks the locus of positions at which the two vessels can meet when using the given speeds. In case the slower vessel makes less than its given speed, the area enclosed by this circle represents the locus of meeting places.

By means of the formula, $S . . . P_1 = \frac{\text{Original separation} \times \text{Speed of S}}{\text{Speed of F plus Speed of S}}$ locate P_1 . In this particular instance, both

the time of earliest meeting and the distance $S . . . P_1$ may be found by using the Logarithmic Scale.

$S . . . P_2$ is then equal to $\frac{\text{Original separation} \times \text{Speed of S}}{\text{Speed of F minus Speed of S}}$.

The center of the circle, O, may be found by bisecting the line $P_1 . . . P_2$ or by the formula,

$S . . . O = \frac{S . . . P_1 \text{ plus } S . . . P_2}{2} \text{ minus } S . . . P_1$.

Tangents drawn from F to the locus circle drawn from O with radius $O . . . P_1$ are the limiting courses that F may take when both ships use their given speeds. Corresponding courses for S are from S to the points of tangency, or $S . . . K'$ and $S . . . K$ respectively.

Answer.—(a) Circumference of a circle of radius 40.0 miles and center 20.0 miles bearing 180° from S. (b) Courses between 150° and 210°, measured clockwise. (c) 090° if F's course is 150°; 270° if F's course is 210°.

NOTE.—In case the speed of F is equal to the speed of S, the radius of the locus circle becomes infinite, and a perpendicular erected at the midpoint of $S . . . F$ becomes the required locus.

In the event that F decides to steer a course between the tangents, such as $F . . . X'$, then S has a choice of 2 courses, $S . . . X$ and $S . . . X'$. In the event that S decides to steer some intermediate course such as $S . . . Y$ while F is moving along the line $F . . . X'$, the time of meeting will be found by dividing the distance $F . . . Y$ by F's speed. S will then use a speed lower than 10.0 knots, found by dividing the distance $S . . . Y$ by the time previously found.

Once the required locus has been found, one ship decides on the course she is to steer and so notifies the other vessel, which must set her course to reach that meeting point.

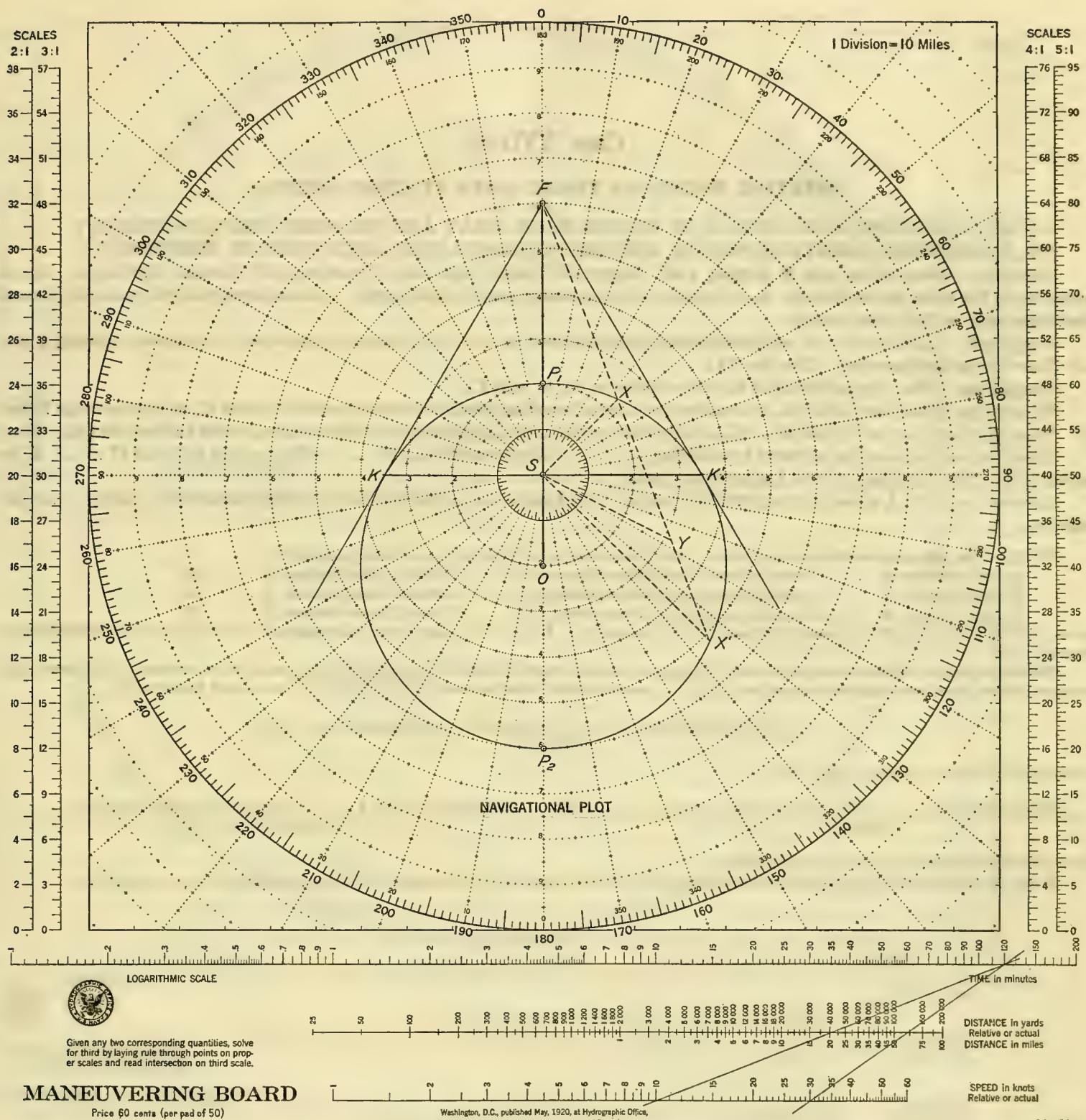


FIGURE 21

Case XVII-B

MEETING POINTS OF THREE SHIPS AT GIVEN SPEEDS

GIVEN: POSITIONS AND SPEEDS OF THREE SHIPS THAT ARE TO MEET SIMULTANEOUSLY.
TO DETERMINE: MEETING POINTS, COURSE FOR EACH SHIP, AND TIME OF MEETING.

Example.—Ship A has ship B bearing 120° , distant 40.0 miles, and ship C bearing 030° , distant 25.0 miles. Speeds available to the ships are as follows: A, 18.0 knots; B, 12.0 knots; and C, 16.0 knots. It is required that the three ships meet simultaneously at their best speeds.

Required.—(a) Location of earliest meeting point relative to A. (b) Course for each ship to earliest meeting place. (c) Time for earliest meeting. (See fig. 22.)

Procedure.—Plot the positions of the three ships at A, B, and C.

By application of case XVII-A, determine the locus of meeting points of ships A and B, A and C, as well as ships B and C. These loci intersect at K and K'. K, being closer to the initial positions of the three ships, is the earliest meeting point.

Time for earliest meeting is found by dividing A K by speed of A, B K by speed of B, or C K by speed of C. This is shown on the Logarithmic Scale.

Answer.—(a) 31.5 miles bearing $088\frac{1}{2}^\circ$ from A. (b) Course for A, $088\frac{1}{2}^\circ$; for B, $351\frac{1}{2}^\circ$; and for C, $137\frac{1}{2}^\circ$. (c) 105 minutes.

NOTE.—The following conditions must obtain if the three units are to be able to meet simultaneously:

A times distance B C must be less than (B times distance A C plus C times distance A B)

B times distance A C must be less than (A times distance B C plus C times distance A B)

C times distance A B must be less than (A times distance B C plus B times distance A C), where A is the speed of A; B, the speed of B; and C is the speed of C. If any of the above three relationships is an equality, there will be but one meeting point.

Should the speeds available and initial relative positions be such that the above relationships do not exist, then the speed of the unit which does not follow the above relationship must be changed. Assuming that A is the speed to be changed, the new speed lies between

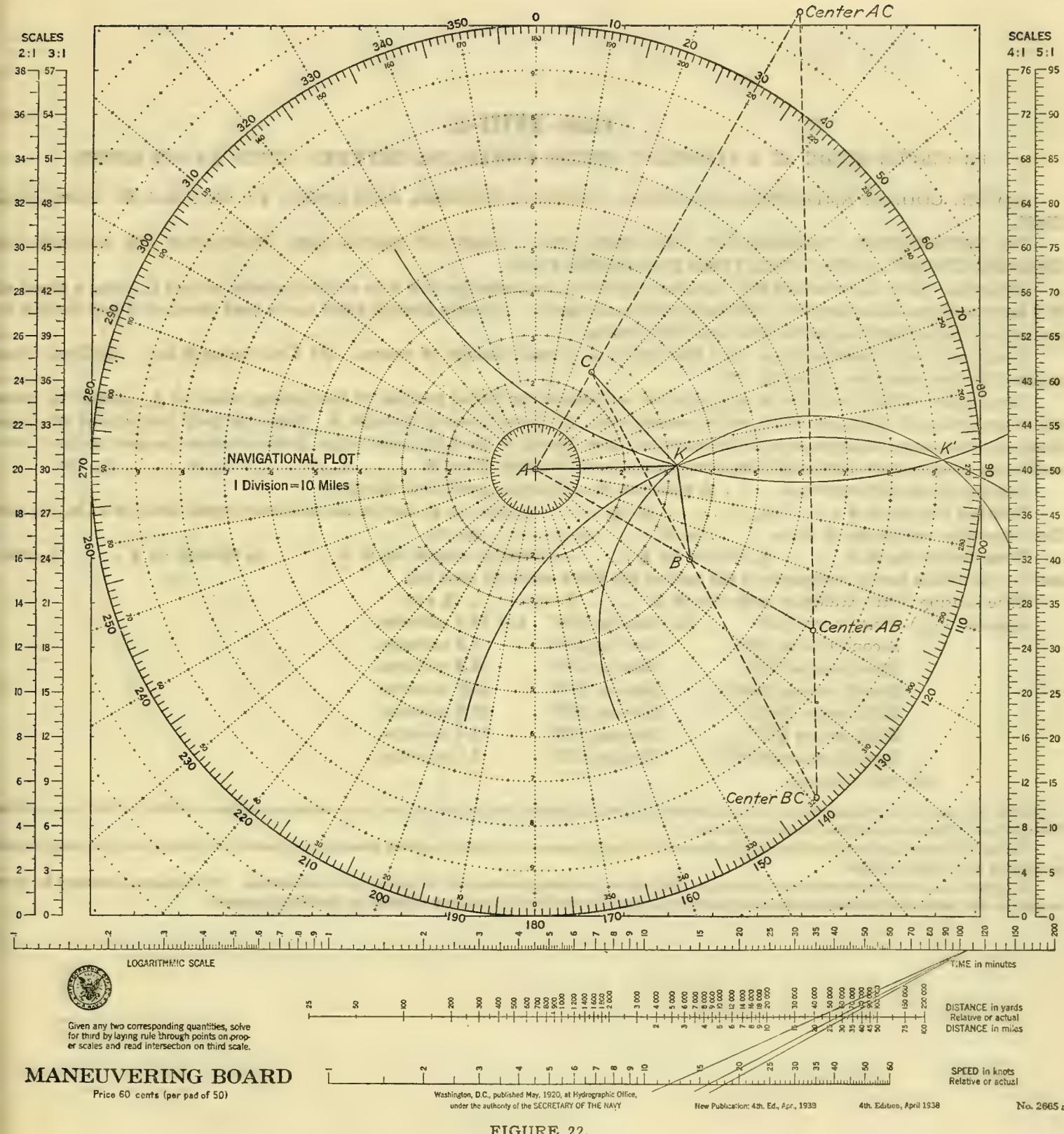
$$A = \frac{(B \text{ times distance } A C) \text{ plus } (C \text{ times distance } A B)}{\text{distance } B C}$$

where there is only one meeting place and

$$A = \frac{(B \text{ times distance } A C) \text{ minus } (C \text{ times distance } A B)}{\text{distance } B C} \text{ or } A = \frac{(C \text{ times distance } A B) \text{ minus } (B \text{ times distance } A C)}{\text{distance } B C}$$

where the relationships permit two meeting places.

In the Navigational Plot, by which this case is solved, it will be noticed that the centers of the locus circles all lie in a straight line.



Case XVIII-A

TO CIRCLE GUIDE AT A CONSTANT SPEED, REMAINING BETWEEN GIVEN RANGE LIMITS

GIVEN: COURSE AND SPEED OF GUIDE, LIMITING RANGES, AND SPEED TO BE USED BY CIRCLING UNIT.

TO DETERMINE: COURSES OF CIRCLING UNIT, TIME ON EACH LEG, BEARINGS ON WHICH TO CHANGE COURSE, AND TOTAL TIME FOR OPERATION.

Example.—Ship G, on course 020° at speed 10.0 knots, requests the ship A to circle clockwise around G, using a speed of 20.0 knots and remaining between 7,000 and 8,000 yards range. A is now located 8,000 yards dead ahead of G and decides to return to its initial position on the final leg of the circling.

Required.—(a) Courses for A. (b) Bearing of G at each change of course. (c) Time on each leg. (d) Total time required for the maneuver. (See fig. 23.)

Procedure.—Plot the Guide at any point G, and locate the initial position of the circling vessel at A. About G, draw the range circles of 7,000 and 8,000 yards, respectively, the latter passing through A. Since the only restrictions on A are those of speed, range, and return to initial position, draw chords of the 8,000-yard range circle which are tangent to the 7,000-yard range circle, except for the last leg, giving Relative Movement Lines $A \dots A_1, A_1 \dots A_2, A_2 \dots A_3$, etc.

Lay out the Guide's vector, $e \dots g$, and draw A's 20.0-knot speed circle about e.

Transfer the slopes $A \dots A_1, A_1 \dots A_2, A_2 \dots A_3$, etc., to g , cutting the 20.0-knot speed circle at a_1, a_2, a_3 , etc. The successive vectors for A are $e \dots a_1, e \dots a_2, e \dots a_3$, etc.

Time on first leg is $A \dots A_1$ divided by $g \dots a_1$; time on second leg is $A_1 \dots A_2$ divided by $g \dots a_2$, etc.; and the total time for the maneuver is the sum of the times spent on each leg.

True bearing of the Guide at each turn is $A \dots G, A_1 \dots G$, etc.

Answer.—(a) First course 113° . (b) Bearing 200° . (c) 10.3 minutes.

Second course 195° . Bearing 258° . 7.8 minutes.

Third course $279\frac{1}{2}^\circ$. Bearing 316° . 9.8 minutes.

Fourth course $340\frac{1}{2}^\circ$. Bearing 014° . 16.8 minutes.

Fifth course $015\frac{1}{2}^\circ$. Bearing 072° . 22.8 minutes.

Sixth course 047° . Bearing 130° . 19.4 minutes.

Seventh course $074\frac{1}{2}^\circ$. Bearing 188° . 3.1 minutes.

(d) 90.7 minutes.

NOTE.—In case the initial position of A is not within the limiting ranges, his first move must be to attain a position on either of the range circles. The solution is then accomplished as shown above.

The normal procedure for A in the example shown is to change course every time the listed bearings are reached, at which time the range should be exactly the outside limits.

This case is most common when the two ships involved are checking bearings for direction finder calibration. If it becomes necessary for A to remain on one particular bearing for checking purposes, A turns to the Guide's course and takes up the Guide's speed.

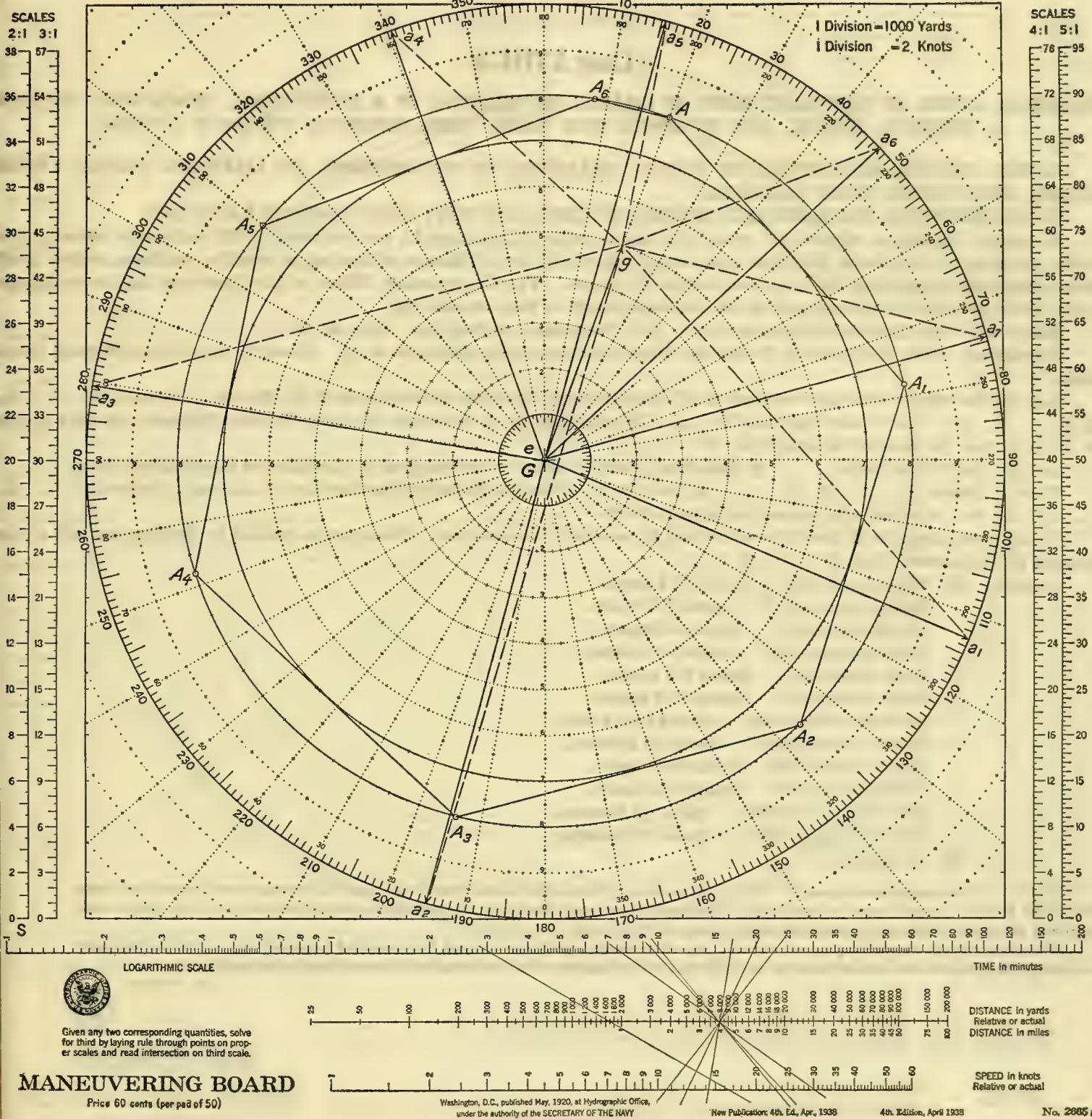


FIGURE 23.

Case XVIII-B

TO CIRCLE GUIDE AT CONSTANT RATE OF CHANGE OF BEARING IN A GIVEN TIME, REMAINING WITHIN A LIMITING RANGE, AND RUNNING FOR SAME TIME INTERVAL ON EACH COURSE

GIVEN: COURSE AND SPEED OF GUIDE, BEARINGS TO BE REACHED AT LIMITING RANGE FROM GUIDE, AND TOTAL TIME ALLOWED.

TO DETERMINE: COURSES AND SPEEDS OF CIRCLING UNIT AND TIME ON EACH LEG.

Example.—Guide on course 020°, speed 10.0 knots, requests Destroyer D, now stationed 8,000 yards dead ahead of the Guide, to circle in a clockwise direction at as nearly a constant rate of change of bearing as possible, remaining within 8,000 yards range and completing the maneuver within 3.0 hours. The Commanding Officer of the Destroyer decides to change course every 30° relative bearing and to run the same length of time on each leg.

Required.—(a) Courses and speeds for D. (b) Time spent on each leg. (See fig. 24.)

Procedure.—Plot the Guide at G and the initial position of the Destroyer at D. About G draw the 8,000-yard range circle.

Lay out the relative bearing lines from G 30° apart, cutting the 8,000-yard circle at D, D_1, D_2 , etc. Connect $D \dots D_1 \dots D_2$, etc. Determine length of $D \dots D_1$, multiply by 12 to obtain the total Relative Distance run, and divide by 3.0 to obtain the Relative Speed of D.

Lay out Guide's vector, $e \dots g$, and about g draw a circle of radius equal to the Relative Speed found above.

Transfer slopes $D \dots D_1, D_1 \dots D_2$, etc., to g , cutting the Relative Speed circle at d_1, d_2 , etc. Vectors $e \dots d_1, e \dots d_2$, etc., represent the courses and speeds for D.

Time on each leg is total time allowed divided by number of courses run. This may be checked by dividing one side of the polygon by the constant rate $g \dots d_1$.

Answer.—(a) First course 066°. Speed 11.2 knots.

Second course 075°. Speed 7.2 knots.

Third course 067°. Speed 3.0 knots.

Fourth course 333°. Speed 3.0 knots.

Fifth course 325°. Speed 7.2 knots.

Sixth course 334°. Speed 11.2 knots.

Seventh course 346½°. Speed 14.5 knots.

Eighth course 359½°. Speed 16.8 knots.

Ninth course 013°. Speed 18.2 knots.

Tenth course 027°. Speed 18.2 knots.

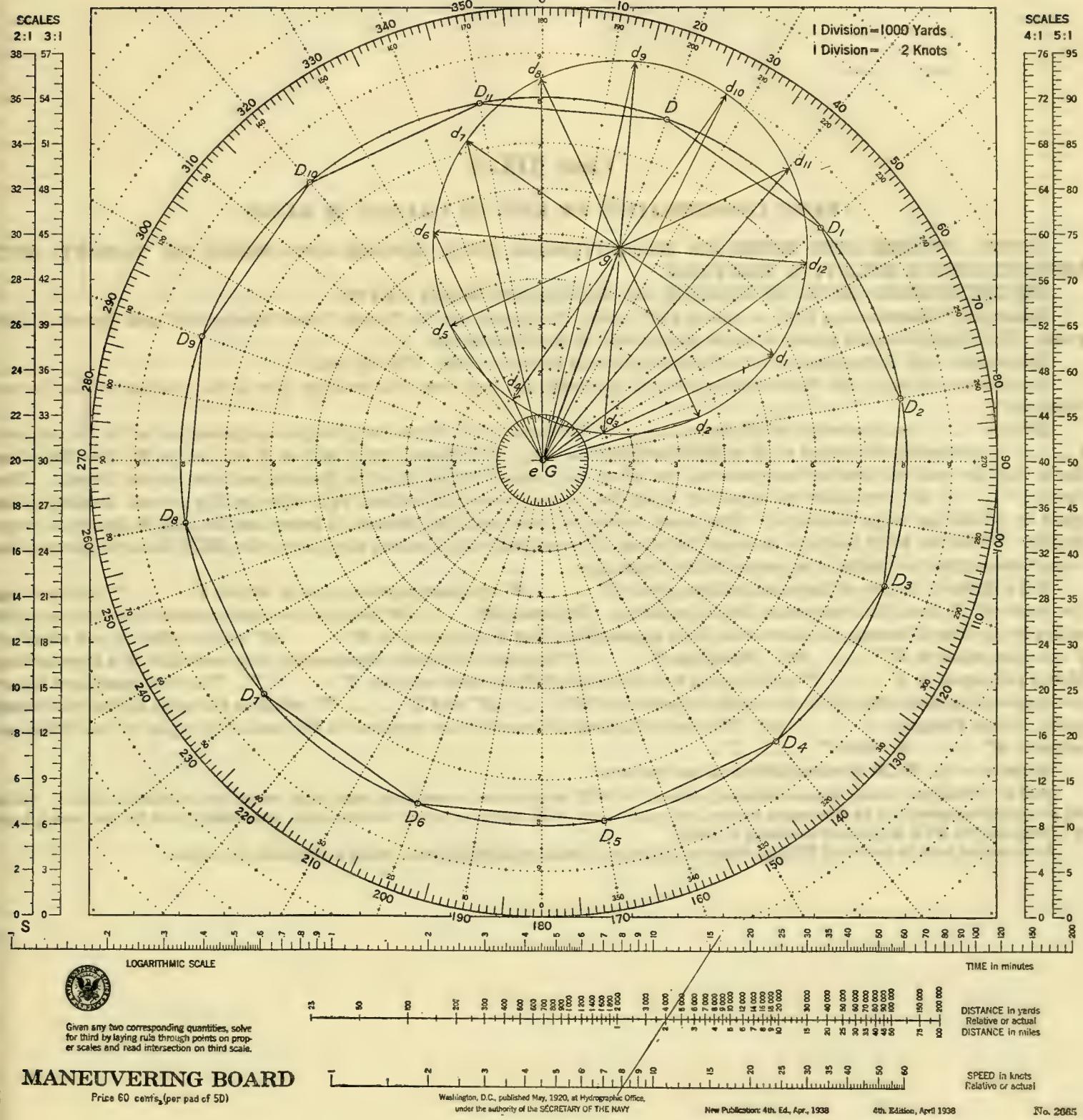
Eleventh course 040½°. Speed 16.8 knots.

Twelfth course 053½°. Speed 14.5 knots.

(b) 15 minutes.

NOTE.—In the conditions imposed in this type of problem, it is possible to construct a regular polygon symmetrical about the course line of the Guide. In such cases it is necessary to obtain courses and speeds for one-half of the vector diagram only, the courses and speeds in the other half being symmetrical with respect to $e \dots g$.

If it is desired that the circling vessel maintain any of the relative positions reached for a specified length of time, it will be necessary to subtract the total time so spent from the time allotted to the maneuver in obtaining the constant rate to be used by D.



Case XIX-A

BASIC CONSIDERATION OF RATE OF CHANGE OF RANGE

GIVEN: COURSE AND SPEED OF MANEUVERING UNIT, COURSE AND SPEED OF TARGET, AND INSTANTANEOUS RELATIVE POSITIONS.

TO DETERMINE: RATE OF CHANGE OF RANGE BETWEEN UNITS.

Example.—Maneuvering Unit on course 020° , speed 11.5 knots, has the Target Vessel bearing 080° and distant 11,500 yards. The Target Vessel is known to be on course 340° , at speed 8.0 knots.

Required.—(a) Instantaneous rate of change of range between units. (See fig. 25.)

Procedure.—Locate the Maneuvering Unit at any point M , and from this position plot in the position of the Target Vessel at T .

Draw $e \dots m$, the vector of M , and $e \dots t$, the vector of T . From the principles explained earlier in this volume, $t \dots m$ represents the travel of M relative to T and $m \dots t$ represents the travel of T relative to M . This Relative Speed may be resolved into two components, one along the present line of Relative Bearing and one normal thereto. The former, $t' \dots m'$, represents the speed at which the two units are approaching each other on the present bearing or the speed by which the range is affected. This speed, converted into yards per minute, is known as the rate of change of range (RCR). When the RCR tends to decrease the range, it is marked with a minus sign; when the RCR tends to increase the range between the two units, it is marked with a plus sign.

For a different picture of the same problem, draw $M \dots M'$ representing the vector of M and $T \dots T'$ representing the vector of T , from their respective initial positions. Resolving $M \dots M'$ into its two components along and at right angles to the line of bearing, it is seen that M is approaching T at a rate equal to $M \dots M''$ and tending to pull ahead of T at a rate equal to $M'' \dots M'$. Resolving $T \dots T'$ into its two similar components, it is seen that T is approaching M at a rate equal to $T \dots T''$ and tending to pull ahead of M at a rate equal to $T'' \dots T'$. The two vessels are therefore approaching each other at a rate equal to the sum of $M \dots M''$ and $T \dots T''$, which is the same as the rate found in the Vector Diagram, $t' \dots m'$. The rate of change of bearing, not required in this set-up, is equal to the difference between $M'' \dots M'$ and $T'' \dots T'$.

Answer.—(a) RCR is minus 238 yards per minute.

NOTE.—When two vessels are on converging courses, the RCR is minus and a maximum when these vessels are on collision courses. When not on collision courses, but on converging courses, the value of the RCR decreases until it becomes zero, after which point the two vessels tend to diverge and the RCR is plus and increasing in value.

A convenient scale to use for all RCR problems is to have one division equal to 3.0 knots, which is 100 yards per minute.

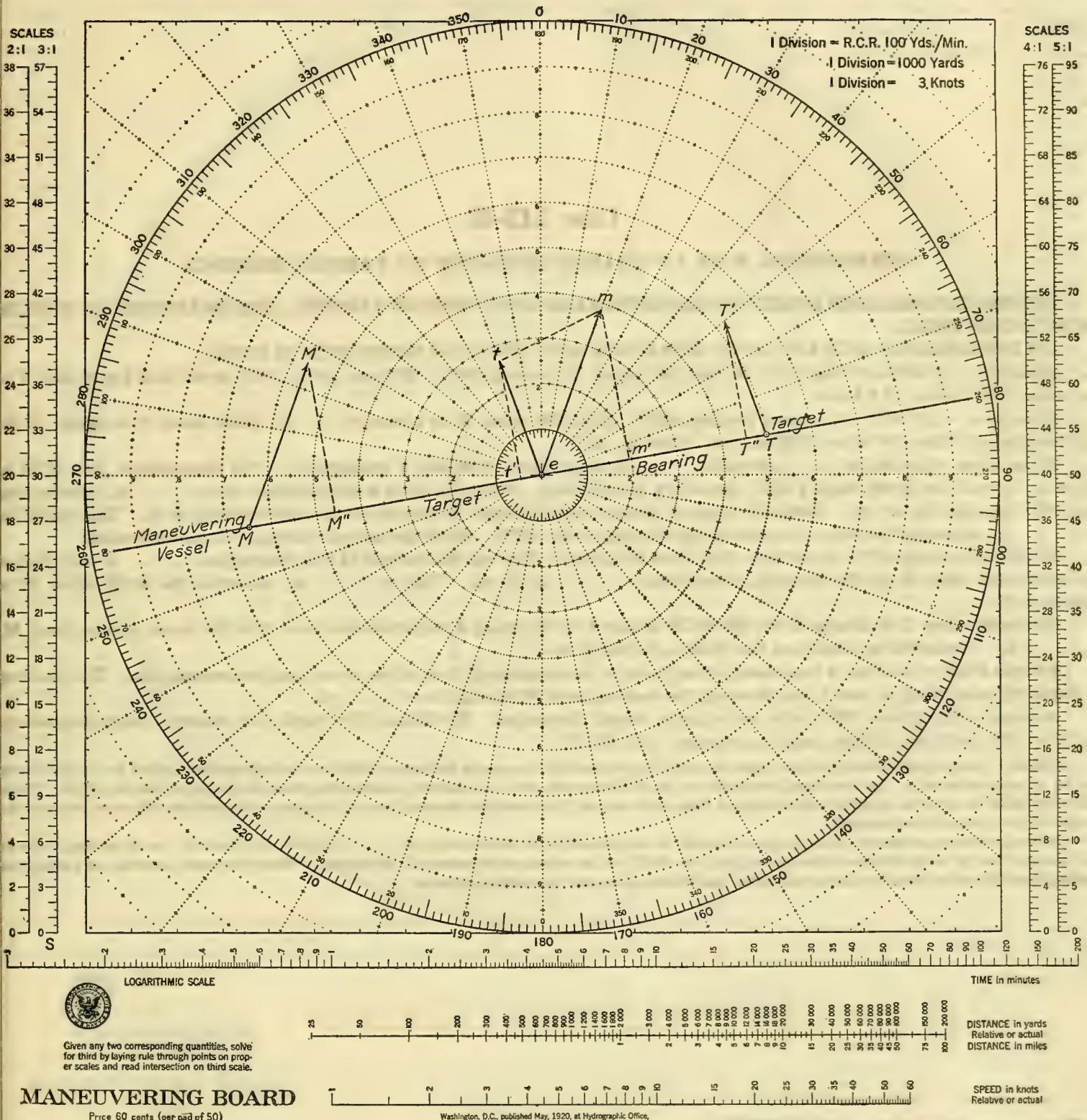


FIGURE 25

Case XIX-B

DETERMINING RATE OF CHANGE OF RANGE ON VARIOUS BEARINGS

GIVEN: COURSE AND SPEED OF MANEUVERING UNIT AND OF TARGET, AND BEARINGS ON WHICH RCR IS REQUIRED.

TO DETERMINE: RCR ON GIVEN BEARINGS AND BEARING WHEN RCR IS ZERO.

Example.—A Maneuvering Unit, M , has the target, T , bearing 090° . M is on course 040° , speed 24.0 knots, and T is on course 200° , speed 18.0 knots.

Required.—(a) RCR on present bearing, 090° . (b) RCR when M is abeam of T . (c) RCR when T is abeam of M . (d) Bearing of T from M when RCR is zero. (See fig. 26.)

Procedure.—Lay out $e \dots m$ and $e \dots t$, the vectors of M and T respectively. For convenience, plot M at e . From e , lay out the given bearing 090° , extending as necessary. From m , drop a perpendicular to $e \dots b_1$, intersecting the 090° bearing line at m_1 . Similarly, from t , drop a perpendicular to this same line, intersecting at t_1 . The length of $t_1 \dots m_1$ measures the rate of closing on this bearing or the RCR. Since the range is closing, it is marked minus.

When M is abeam of T , it will bear 290° from the latter. Lay out the reverse of this bearing as $e \dots b_2$ and drop a perpendicular from M to this bearing, intersecting $e \dots b_2$ at m_2 . Then $e \dots m_2$ measures the RCR when M is abeam of T .

Similarly, when T is abeam of M , the RCR is found by dropping a perpendicular from t to the beam bearing from M , $e \dots b_3$. Under these conditions the RCR is indicated by $e \dots t_2$.

For the RCR to be zero, it is necessary that there be no component from either m or t on the bearing line. This bearing is found by connecting m and t , and drawing the bearing from M normal to $m \dots t$.

Answer.—(a) Minus 720 yards per minute, range decreasing. (b) Minus 274 yards per minute, range decreasing. (c) Plus 205 yards per minute, range increasing. (d) $121\frac{1}{2}^\circ$.

NOTE.—The perpendiculars dropped from m and t to the bearing lines make intercepts which are added together to find the RCR if they are on opposite sides of e . In case these intercepts are on the same side of e , the shorter is subtracted from the longer to find the RCR. Thus, when the RCR is zero, the perpendiculars dropped from both m and t intersect the bearing line at the same point and the difference between the length of the intercepts becomes zero.

As long as the target bearing is on the approach side of the normal to the relative line, $m \dots t$, (when the bearing line is moving towards the normal from M to the Relative Movement Line), the RCR will be minus or the range will be closing. When the target bearing has passed this normal from M to the Relative Movement Line, the RCR is plus and the range is opening.

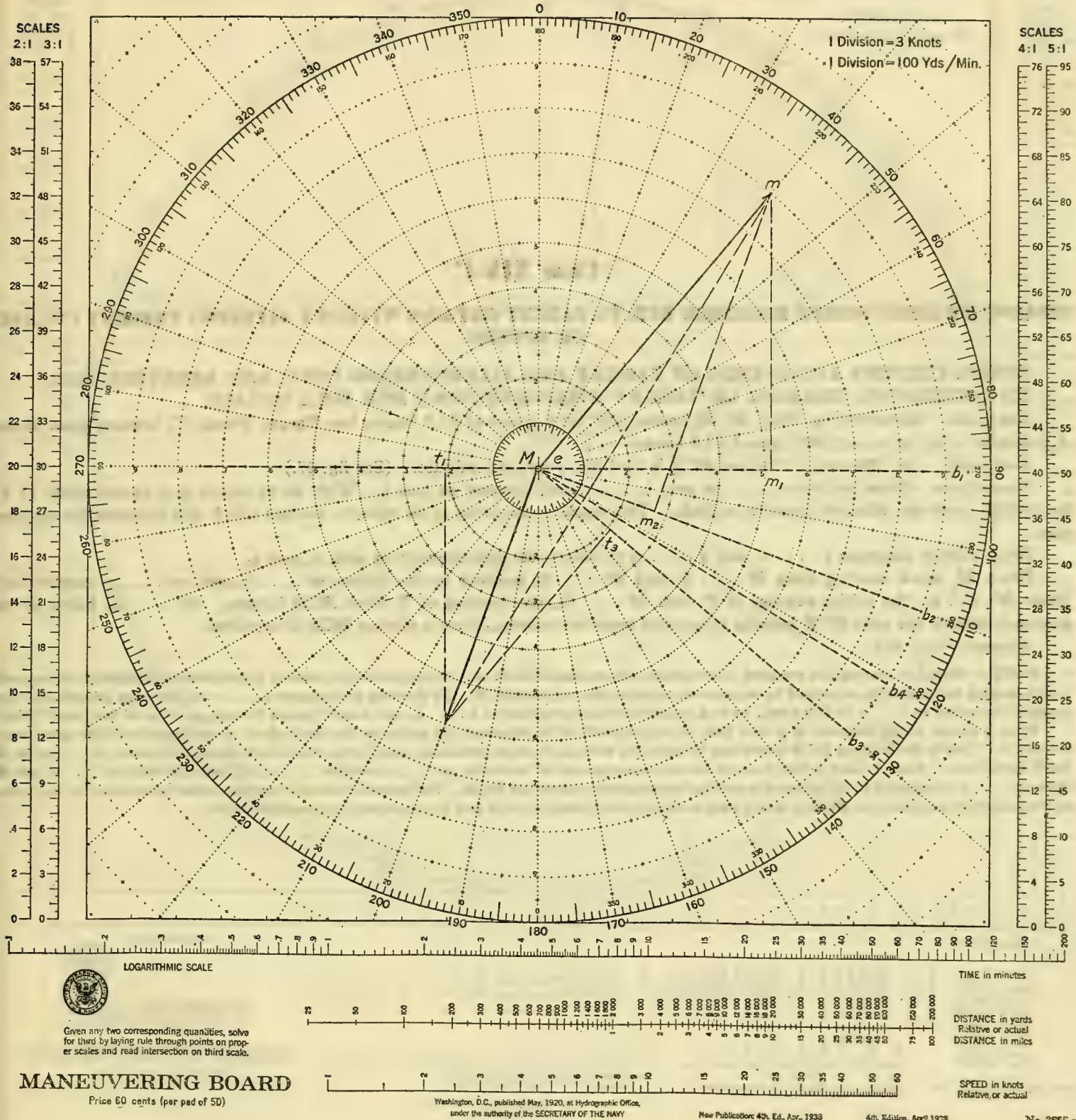


FIGURE 26.

Case XIX-C

FINDING BEARING WHERE REQUIRED RCR TO TARGET OBTAINS WITHOUT ALTERING PRESENT COURSES OR SPEEDS

GIVEN: COURSES AND SPEEDS OF TARGET AND MANEUVERING UNIT, AND REQUIRED RCR.

TO DETERMINE: BEARING OF TARGET WHEN REQUIRED RCR WILL OBTAIN.

Example.—Maneuvering Unit, M , on course 035° at a speed of 21.0 knots, has Target Vessel, T , bearing dead ahead. T is known to be on course 080° , speed 12.0 knots.

Required.—(a) Bearing of T when RCR is $(-)$ 300 yards per minute. (See fig. 27.)

Procedure.—Draw vectors $e \dots m$ and $e \dots t$ and connect m and t . With m as center and radius equal to 9.0 knots (300 yards per minute) describe a circle. This circle is the locus of all relative vectors which will produce the required rate.

From t draw tangents $t \dots r_3$ and $t \dots r_4$ to this circle and connect m with r_3 and r_4 .

From M , draw bearing lines $M \dots b_3$ and $M \dots b_4$ parallel to the slopes $m \dots r_3$ and $m \dots r_4$ respectively. Draw $M \dots b_1$, the initial bearing of T , and $M \dots b_2$, the bearing of T when RCR is zero. $M \dots b_3$, being on the approach side of the zero RCR bearing line is the required bearing, since a closing RCR is required.

Answer.—(a) 054° .

Note.—Since the bearing line required is one which will produce a RCR of $(-)$ 300 yards per minute, perpendiculars dropped from m and t to this bearing line should be separated by an amount equal to this required RCR. By drawing the tangent $t \dots r$ to the locus circle from t and drawing the radius $m \dots r_3$ to this point, we find the radius which is normal to $t \dots r_3$, and since it passes through m , it is of the proper length.

From a glance at the diagram, it is seen that as the required RCR increases, the radius of the circle about m increases until the circle passes through t . This is the limiting RCR which can be attained with the vectors presented, and in the illustrated example becomes $m \dots t$ or 502 yards per minute. Since t itself is then located on the circumference of the circle, $m \dots r_3$ and $m \dots r_4$ will coincide with $m \dots t$, and the slope $m \dots t$ transferred to M indicates a collision bearing with a constant RCR. The two units would therefore either be heading for each other at the maximum and constant RCR or would have started from a common point and be separating at a constant RCR.

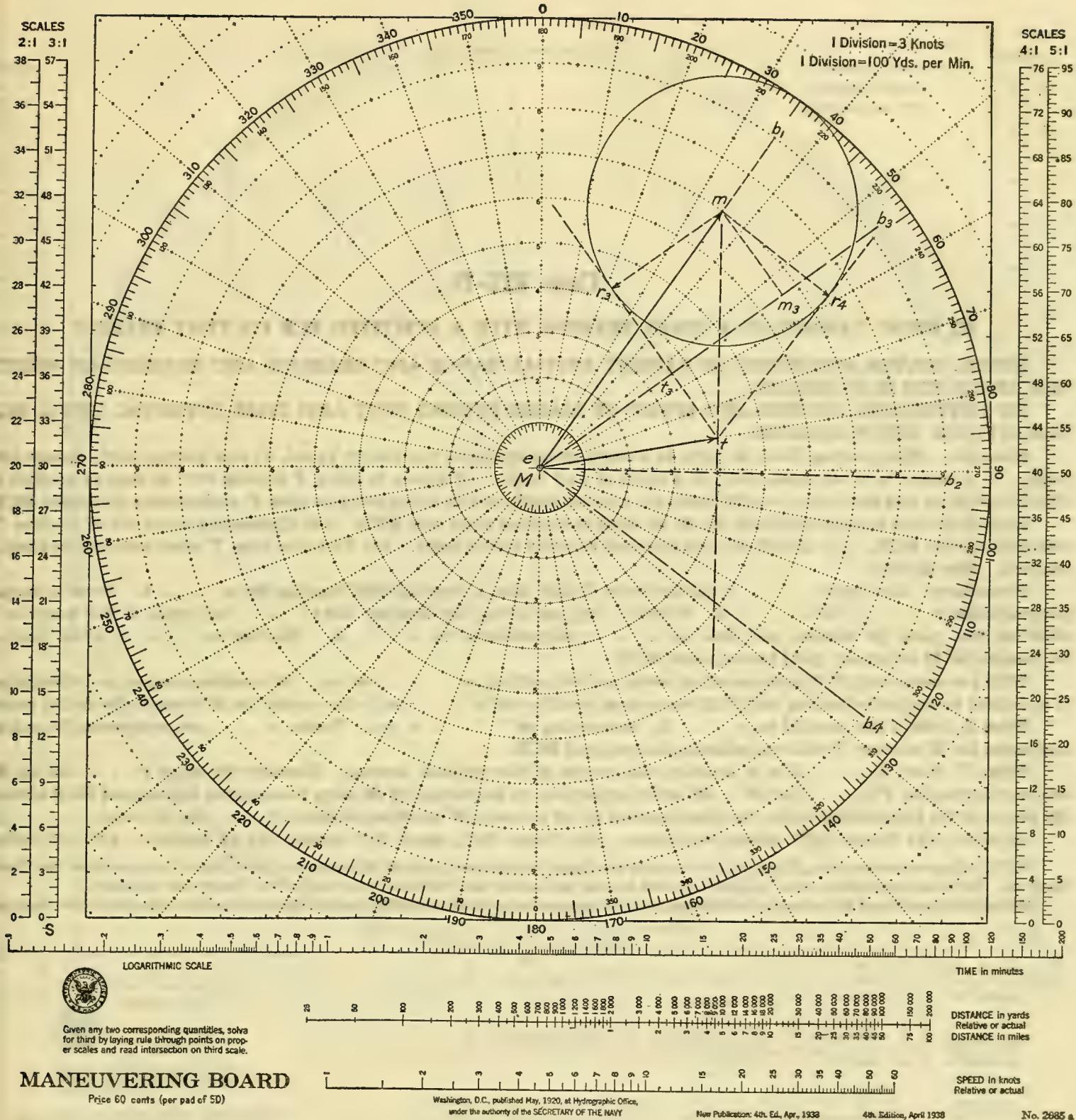


FIGURE 27.

Case XIX-D

TO BRING TARGET ON A GIVEN BEARING WITH A SPECIFIED RCR ON THAT BEARING

GIVEN: COURSE AND SPEED OF TARGET, INITIAL RANGE AND BEARING, AND BEARING ON WHICH SPECIFIED RCR IS TO OBTAIN.

TO DETERMINE: COURSE AND SPEED OF MANEUVERING UNIT AND TIME INTERVAL UNTIL RCR CONDITIONS ARE FULFILLED.

Example.—Maneuvering Unit, M , with 24.0 knots speed available, has Target Vessel, T , now bearing 320° , distant 10.0 miles. T is known to be on course 340° at a speed of 15.0 knots. M wishes to bring T bearing 000° as soon as possible so that the RCR on this bearing is $(-)$ 200 yards per minute, and from this initial point to close T , maintaining this same RCR.

Required.—(a) Course and speed for M to reach this initial point and RCR. (b) Course and speed for M to close T , maintaining this RCR. (c) Length of time required to reach initial point. (d) Distance from T when required bearing is reached. (See fig. 28.)

Procedure.—Lay out $e \dots t$, the vector of T , and from e draw the 000° bearing line $e \dots b$. From t , drop a perpendicular $t \dots t'$ to $e \dots b$. From t' , lay out along the bearing line $t' \dots m'$ equal to 6.0 knots or a RCR of 200 yards per minute, and draw $m' \dots r$ perpendicular to $e \dots b$. The line $m' \dots r$ is the locus of all courses for M which will yield the required RCR.

With e as center and 24.0 knots as radius, swing a circle, cutting $m' \dots r$ at m_1 . M 's maximum speed is utilized since it is desired to reach the bearing and RCR as soon as possible and $e \dots m_1$ is the vector for M to reach the initial point.

From t , draw a line parallel to $e \dots b$, intersecting $m' \dots r$ at m_2 . Vector $e \dots m_2$ represents the course and speed for M to close T while maintaining the required RCR.

From T , draw $T \dots X$ in a direction the reverse of the required bearing. Transfer the slope $t \dots m_1$ to M , intersecting the line $T \dots X$ at M' . M' is the initial point to be reached by M when the required bearing and RCR obtain. By means of the Logarithmic Scale, the time required for M to reach M' at Relative Speed $t \dots m_1$ is found.

Answer.—(a) Course 327° , speed 24.0 knots. (b) Course 346° , speed 20.7 knots. (c) 48 minutes. (d) 2.85 miles.

NOTE.—In problems involving RCR only, the range to the Target is immaterial, since RCR is purely a velocity function found from the Vector Diagram. (In the above problem, time to reach a desired bearing was added; and, therefore, the range of the Target was necessary.)

Although the position of M , in the examples shown, has been assumed at e , the Relative Plot does not have to coincide with the Vector Diagram.

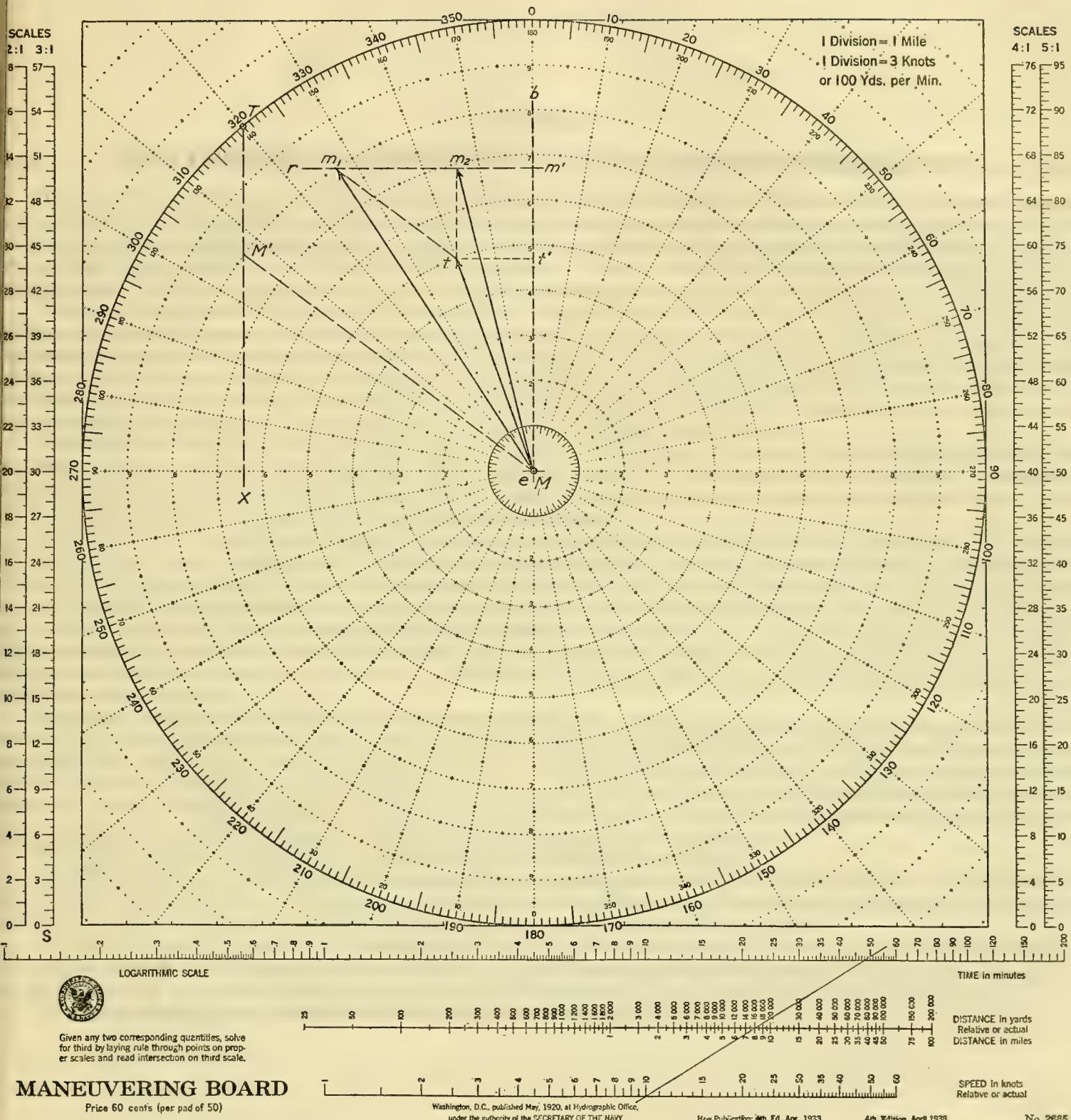


FIGURE 28.

SECTION II

TWO COURSES FOR GUIDE, SINGLE COURSE FOR MANEUVERING UNIT

The cases thus far discussed have comprised either those in which both Guide and Maneuvering Unit maintain constant courses and speeds throughout the problem, or else they have involved successive applications of cases of this type. In this section we deal with situations which require a change of course or speed or both on the part of the Guide while the Maneuvering Unit is proceeding from one Relative Position to another. When the total time for the maneuver is given, as well as the exact time the Guide changes course or speed or both, the obvious and most rapid solution is a simple Navigational Plot.

When the Navigational Plot may not be thus employed, the problem may be solved by so manipulating or distorting the Relative Movement Diagram that the cases resolve themselves into the Single Vector Cases discussed in section I. This distortion is required because the Guide does not maintain its course and speed throughout the problem, and consists in employing an imaginary or Fictitious Guide which *does* maintain its course and speed throughout the problem. The use of a Fictitious Guide should be readily understood by any practical navigator who has had to compute the net run of a vessel in Traverse Sailing, for finding the Course and Distance made good between Fixes of Position.

This Fictitious Guide may be considered as accompanying the real or actual Guide on either its first or its second leg, depending upon the conditions of the problem. The course and speed of the Fictitious Guide will therefore be represented by one of the vectors of the real Guide. Since the Fictitious Guide uses this vector throughout the problem, it may be employed in exactly the same manner as previously described in section I.

In the Relative Plot, positions are located with reference to the *real* Guide by the statement of the problem. If, however, the problem requires the employment of a Fictitious Guide for purposes of solution, it is essential that all positions in the Relative Plot be *referred to the Fictitious Guide* (reorienting as necessary) *and not to the actual Guide*. As long as the real Guide and the Fictitious Guide are *together*, any position plotted relative to the one is plotted relative to the other; but this relationship does not hold when the real Guide and the Fictitious Guide are *not together*. In order that we may keep our Fictitious Guide stationary in the Relative Plot, it is necessary to offset one of the positions of the Maneuvering Unit by the amount this position would be shifted with respect to the Fictitious Guide during the time that the Fictitious Guide and the real Guide are not in company.

In general, the initial set up of the Vector Diagram comprises the vectors representing the two courses and speeds of the actual Guide, one of which vectors will therefore also be the vector of the Fictitious Guide. The Relative Plot initial set up consists of the real Guide and the Fictitious Guide located together at any convenient point, and the initial and the final relative positions of the Maneuvering Unit plotted with reference to the real Guide. One of these Relative Positions is next offset the proper amount to relocate it in respect to the Fictitious Guide. The other relative position remains untouched since it is plotted for the time when the real Guide and the Fictitious Guide are together.

The following rules govern cases involving employment of the Fictitious Guide:

(1) *When the time of departure of the Maneuvering Unit is known*, the vector of the Fictitious Guide coincides with the *second* vector of the real Guide. The *initial* relative position of the Maneuvering Unit is therefore offset by the relative run of the real Guide with respect to the Fictitious Guide, while the real Guide is on its *first* course.

(2) *When the time of arrival of the Maneuvering Unit at its final relative position is known*, the vector of the Fictitious Guide coincides with the *first* vector of the real Guide. The *final* relative position of the Maneuvering Unit is therefore offset by the relative run of the real Guide with respect to the Fictitious Guide, while the real Guide is on its *second* course.

(3) *The direction of offset is always toward the second vector of the real Guide.*

Case XX

TO PROCEED FROM ONE RELATIVE POSITION TO ANOTHER AT KNOWN SPEED, ARRIVING AT GIVEN TIME, GUIDE CHANGING COURSE DURING OPERATION

GIVEN: COURSES AND SPEEDS OF GUIDE AND TIME AT WHICH GUIDE WILL CHANGE COURSE, INITIAL AND FINAL RELATIVE POSITIONS, SPEED OF MANEUVERING UNIT, AND TIME OF ARRIVAL AT FINAL POSITION.

TO DETERMINE: COURSE OF MANEUVERING UNIT AND TIME OF LEAVING INITIAL RELATIVE POSITION.

Example.—At 0800 Guide is on course 140° , speed 14.0 knots, and will change to course 070° , speed 16.0 knots, at 0930. Ship M, now stationed 8.0 miles on the port beam of the Guide, is ordered to take station 12.0 miles on the starboard beam of the Guide, arriving at 1200, time of departure optional. M decides to maintain present station as long as possible so as to carry out his orders at 14.0 knots.

Required.—(a) Course of M. (b) Time M leaves present station. (c) Minimum speed and corresponding course available to M. (See fig. 29.)

Procedure.—Lay out $e \dots g_1$ and $e \dots g_2$, the first and second vectors of the Guide. Join g_1 and g_2 .

Plot the position of the Guide at any point G. Locate M_1 , the initial position of M, which is 8.0 miles bearing 050° from G. In a similar manner locate M_2 bearing 160° and distant 12.0 miles from G.

From the rules listed on the preceding page, the vector of the Fictitious Guide coincides with the vector of G's first leg, and the final position of M must be offset for the time that the real Guide and the Fictitious Guide are not together.

Offset M_2 in the direction parallel to $gf \dots g_2$, a distance equal to the Relative Run for 150 minutes at Relative Speed $gf \dots g_2$, as found from the Logarithmic Scale, locating M_2' . Join M_1 and M_2' , and transfer the slope of $M_1 \dots M_2'$ to gf cutting the 14.0 knot circle at m. The course for M at 14.0 knots is indicated by $e \dots m$.

Using the Relative Speed $gf \dots m$ and the Relative Distance $M_1 \dots M_2'$, the time required for M to complete the maneuver at 14.0 knots is found. This time, subtracted from 1200, gives the time at which M should first head for his final position.

If the minimum speed is to be used, M must start at once. His course and speed would either be found by a simple Navigational Plot, or, using the offset position previously plotted, the Relative Speed would be the distance $M_1 \dots M_2'$ divided by the 4.0 hours available. This Relative Speed, $gf \dots m'$, indicates that the minimum speed and the corresponding course is given by the vector $e \dots m'$.

Answer.—(a) 098° . (b) 163 minutes before 1200 or at 0917. (c) 13.2 knots on course 111° .

NOTE.—The procedure would be the same were the course and not the speed specified, the determining intersection being that of the slope from gf parallel to $M_1 \dots M_2'$ with the specified course line.

In case $e \dots m'$ is not normal to $gf \dots m$, it is possible to use a lower speed where these conditions obtain. However, the problem will have to start earlier than 0800 in order to utilize this lower speed.

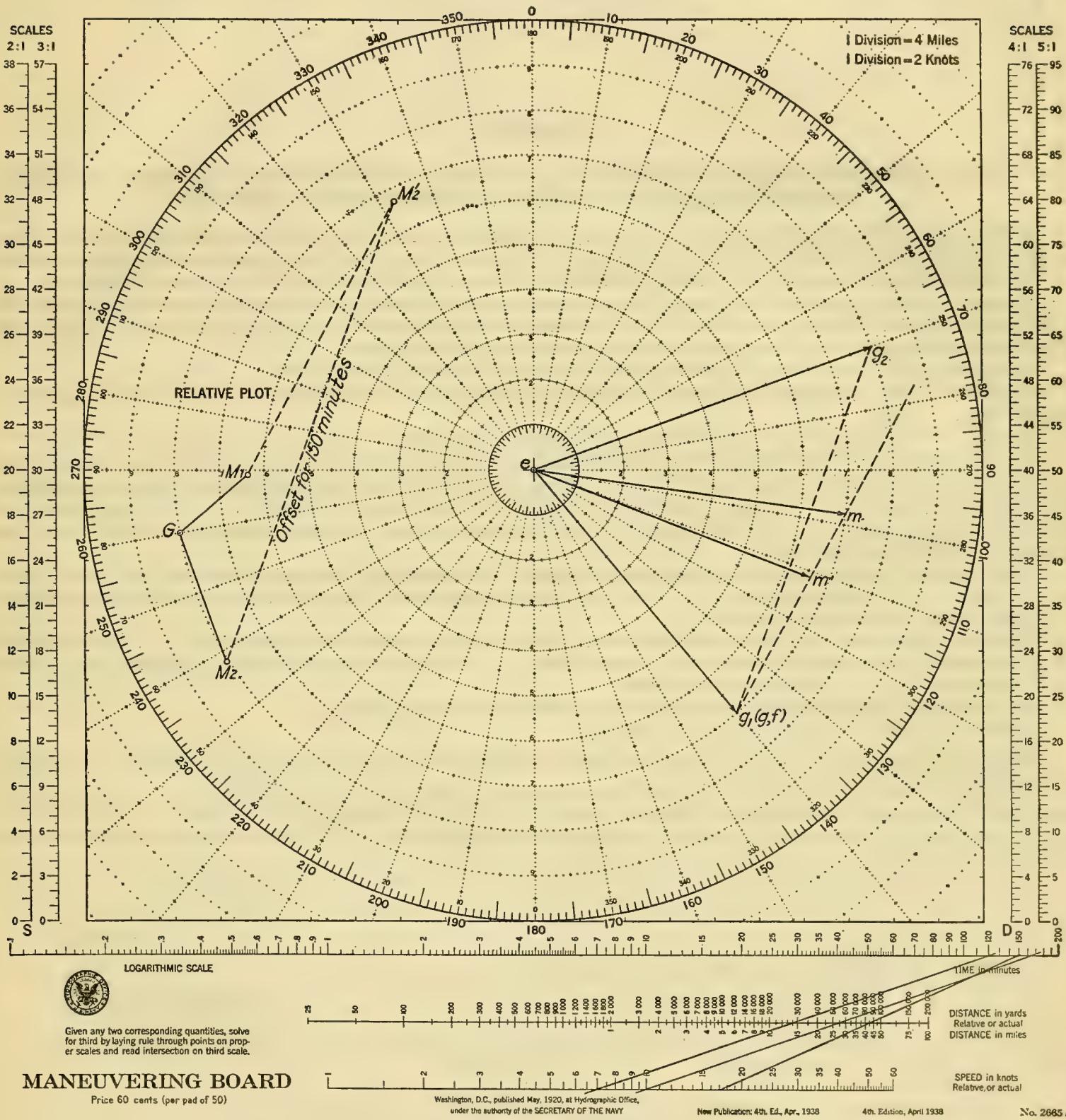


FIGURE 29.

Case XXI

TO PROCEED FROM ONE RELATIVE POSITION TO ANOTHER AT KNOWN SPEED, STARTING AT KNOWN TIME, GUIDE CHANGING COURSE AND SPEED DURING OPERATION

GIVEN: COURSES AND SPEEDS OF GUIDE AND TIME ON FIRST LEG, INITIAL AND FINAL RELATIVE POSITIONS OF MANEUVERING UNIT, SPEED OF MANEUVERING UNIT, AND TIME MANEUVER STARTS.

TO DETERMINE: COURSE OF MANEUVERING UNIT AND TIME TO ARRIVE IN FINAL POSITION.

Example.—Guide is on course 320° , speed 15.0 knots, and has signaled that at 1400 her course and speed will be changed to 280° and 12.0 knots. Ship M, now located 18.0 miles broad on the port beam of the Guide, receives orders at 1130 to take position 10.0 miles bearing 055° from the Guide as soon as possible using 15.0 knots.

Required.—(a) Course for M. (b) Time required to reach final position. (See fig. 30.)

Procedure.—Plot the Guide at any point G, and locate the initial and final positions of M at M_1 and M_2 , respectively. Measure $M_1 \dots M_2$.

Draw $e \dots g_1$ and $e \dots g_2$, the vectors of G's first and second legs.

A rapid inspection of the plotted positions will indicate that it will be impossible for M to reach its new station in the 2.5 hours that G is on the first leg, since M is held to the same speed as G and must shift from 18.0 miles on one beam to 10.0 miles slightly abaft the other beam. This may be checked by transferring the slope $M_1 \dots M_2$ to g_1 , cutting the 15.0 knot circle at g_1 and m' . Relative Speed $g_1 \dots m'$ is too low to permit the change of station while G is on her first leg, so a Fictitious Guide must be used.

Following the rules formulated in the introductory pages of this section, the Fictitious Guide takes the second vector of G, so that $e \dots gf$ coincides with $e \dots g_2$, and the initial position of M is offset for the time that the real Guide and the Fictitious Guide are not together.

By means of the Logarithmic Scale, using the time G is on the first leg, and the Relative Speed $g_1 \dots gf$, the distance M_1 is to be offset is obtained, thus locating M_1' . Connect $M_1' \dots M_2$ and transfer this slope to g_2 , cutting the 15.0 knot speed circle at m . $e \dots m$ is the course for M.

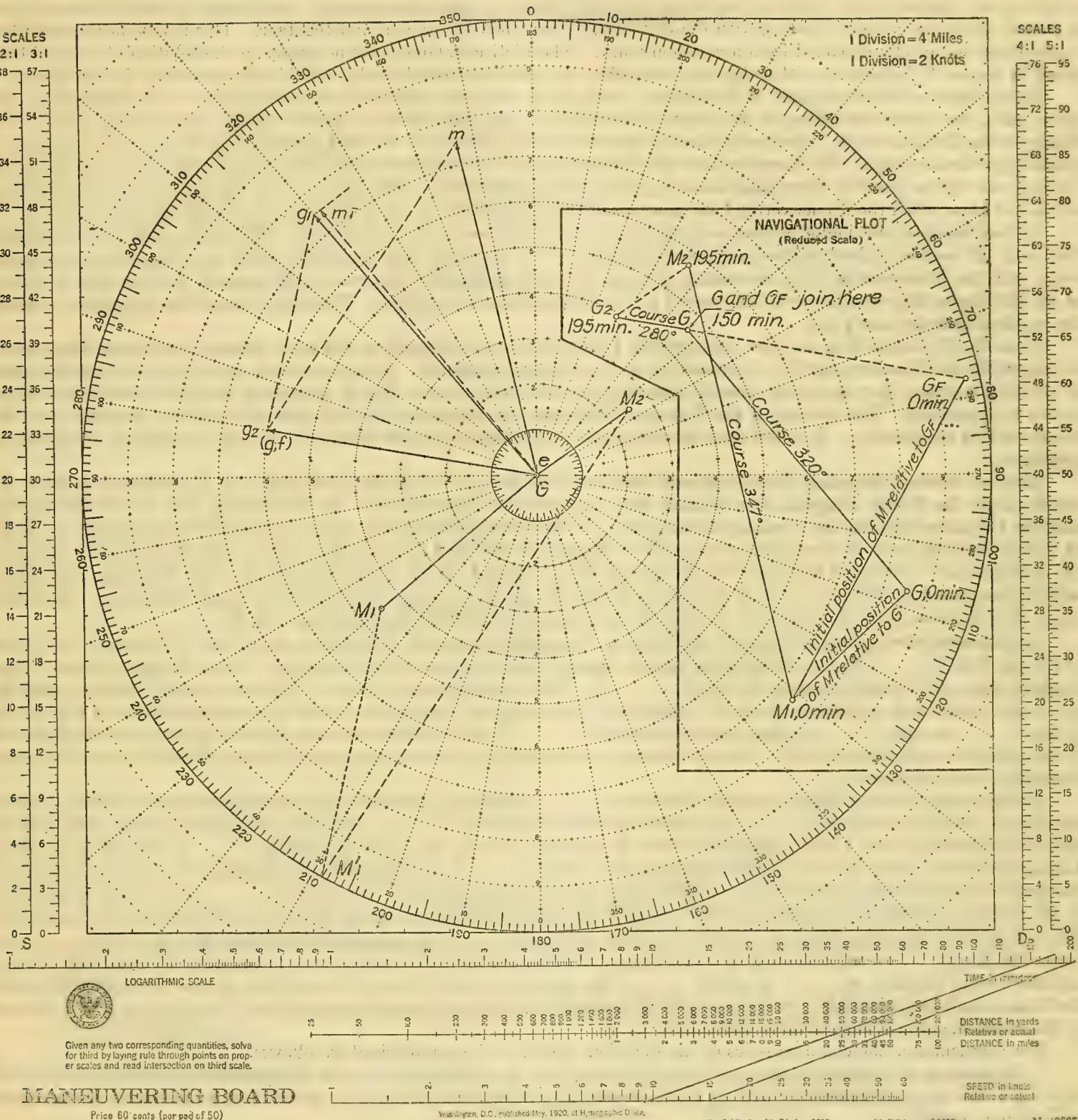
By means of the Logarithmic Scale, the time required for the maneuver is found.

Answer.—(a) 347° . (b) 195 minutes or 3.25 hours.

NOTE.—Had minimum speed for M been a requirement instead of minimum time at 15.0 knots, the vector for M would have been found by dropping a perpendicular from e to the slope $g_2 \dots m$.

Had the speed for M been between $e \dots g_2$ and the minimum speed as explained above, and M was not required to be in position as soon as possible, two solutions would result through the transferred slope $M_1' \dots M_2$ crossing the specified speed circle twice.

A Navigational Plot, to reduced scale, is inserted on the diagram to give a graphic picture of the tracks of the real Guide, the Fictitious Guide, and the Maneuvering Unit.



SECTION III

TWO COURSES FOR MANEUVERING UNIT, SINGLE COURSE FOR GUIDE (THE FICTITIOUS SHIP)

In section II we dealt with a Fictitious Guide which, while maintaining a single course and speed, accompanied the actual Guide for an indefinite time while the latter was on either its first or its second course. One feature of this Fictitious Guide is that it maintains constant bearing with the real Guide, even when the two are not together. This is true because the two units either met or else departed from a common point. The Fictitious Guide was necessary in the solution of the problems listed in section II because we either did not know when the problem started or else we did not know when it terminated.

In the cases to be investigated in this section, we do know when the problem starts as well as when it terminates. We know, also, the course and speed of the Guide and the initial and final relative positions of the Maneuvering Unit. The Maneuvering Unit, however, is not going directly from the initial position to the final position, but it is going to some intermediate position en route. We therefore have two Lines of Relative Movement to consider instead of the one usually involved. In some cases these Relative Movement Lines may be specified chart lines. Although we know the total time of the problem, we do not know when the Maneuvering Unit must reach the intermediate point until we have partially solved the problem.

To facilitate the solution of problems of this type, another fictitious unit is introduced. This unit is called the Fictitious Ship and will be used in both surface and aerial problems. The Fictitious Ship leaves the initial relative position simultaneously with the Maneuvering Unit, proceeding directly for the final relative position at such a speed that the time of arrival of the Fictitious Ship coincides with the time of arrival of the Maneuvering Unit. The latter, meanwhile, has passed through the required intermediate point.

Since the Fictitious Ship and the Maneuvering Unit leave the initial point at the same instant, they maintain constant bearing while the Maneuvering Unit is proceeding to the intermediate point. Also, since these two units come together again at the final position, they maintain constant bearing while the Maneuvering Unit is proceeding from the intermediate point to the final position. The bearing of the Fictitious Ship from the Maneuvering Unit (and the reverse), therefore does not change during the problem. It is this feature which gives the Fictitious Ship its particular value in the solution of two-course problems.

Another feature, intimately connected with the Fictitious Ship, is called the Time Line and is illustrated in figure 31, which shows the Vector Diagram, the Relative Plot, and the Navigational Plot. In this sketch, both the Fictitious Ship and the Maneuvering Unit are originally at the point A . The Maneuvering Unit steers course 050° for 2.0 hours at a speed of 12.0 knots until point P is reached, when course is changed to 320° and speed to 18.0 knots. This latter course and speed are continued for 1.0 hour, at the end of which time the Maneuvering Unit has arrived at point B . The Fictitious Ship, which will be designated as F leaves point A and heads directly for point B , using course 013° and speed 10.0 knots. It will be noted that F leaves point A and arrives at point B simultaneously with the Maneuvering Unit M .

As M proceeds from point A to point P in 2.0 hours, F runs from A to point F_1 ; therefore, at the end of 2.0 hours, the bearing of F from M is $P \dots F_1$, and this bearing has not changed since the two units left point A . When M turns at point P , and heads for point B , F continues along the line $F_1 \dots B$. The bearing between M and F does not change, therefore, and the two units would be in collision at point B . Using the Vector Diagram, where the vectors for the first and second legs run by M are $e \dots m_1$ and $e \dots m_2$, respectively, the slope $P \dots F_1$ when transferred to m_1 is found to also pass through m_2 and f , the extremity of the vector for F . This, of course, is as it should be, because the bearing between F and M does not change during the entire problem.

At the end of 2.0 hours, M has departed a distance $F_1 \dots P$ from F . This distance divided by the time involved yields a rate which, when laid off on $m_1 \dots m_2$, is found to coincide with $m_1 \dots f$. Since M must again coincide with F at the end of the problem, or in 1 hour, the relative rate of approach between M and F must be $P \dots F_1$ divided by 1.0 hour. Laying off this rate from m_2 , we see that it indicates the vector $m_2 \dots f$. From this, it is apparent that the line connecting the two vectors of M also includes the end of F 's vector, and that the rates $m_1 \dots f$ and $m_2 \dots f$ are directly proportional to the rates of departure and approach of the two units and are inversely proportional to the times of departure and approach. The two units are departing from one another while M is on course $e \dots m_1$ and approaching while M is on course $e \dots m_2$, hence the line $m_1 \dots m_2$ is divided by f into segments which are inversely proportional to the times M spends on the first and the second courses. $m_1 \dots f$ and $m_2 \dots f$ are relative vectors because the two units are on the courses indicated for M and for F simultaneously. $m_1 \dots m_2$ is not a relative vector, but is what is described as a Time Line, so called because of the above-mentioned time division determined by f .

For convenience, the characteristics of the Fictitious Ship may be summarized as follows:

1. It proceeds at constant speed and on a single course, from the initial position to the final position, leaving the former and arriving at the latter coincidentally with its reference ship, or unit, which reference ship or unit meanwhile passes through some intermediate point.

2. It maintains constant bearing with its reference unit throughout the operation.

3. The Fictitious Vector divides the line connecting the two vectors of the Reference or Maneuvering Unit into segments inversely proportional to the times spent by the Maneuvering Unit on its two courses.

4. The Fictitious Ship can be used only for the solution of problems in which both time of start and time of finish are known.

The above characteristics of the Fictitious Ship will become familiar during the solution of the two-course problems presented in illustrating the cases for this section. A number of scouting problems are included, involving both surface craft and aircraft. During these problems when *minimum speed* is required, the *earlier* the Maneuvering Unit leaves its initial position the *lower* will be the speed necessary. Also, to use *minimum speed*, the *same* speed must be used on *both legs*.

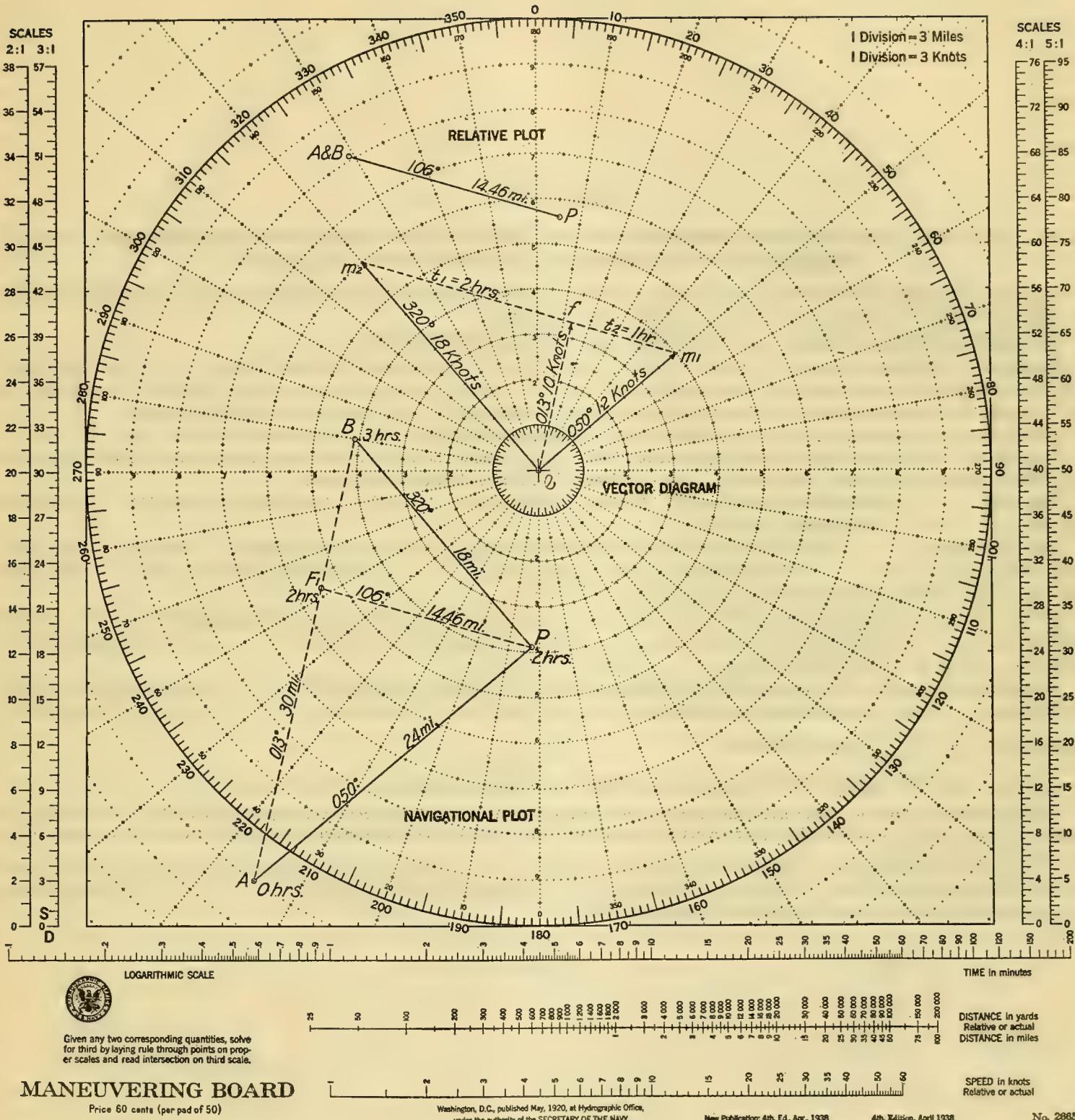


FIGURE 31.

Case XXII

TO PROCEED FROM ONE RELATIVE POSITION TO ANOTHER IN GIVEN TIME AT MINIMUM SPEED, PASSING THROUGH AN INTERMEDIATE RELATIVE POINT EN ROUTE

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL, INTERMEDIATE, AND FINAL RELATIVE POSITIONS OF MANEUVERING UNIT, AND TOTAL TIME INTERVAL.

TO DETERMINE: COURSES AND SPEEDS OF MANEUVERING UNIT.

Example.—Guide on course 040° , speed 12.0 knots, has ship M now stationed 8.0 miles broad on her starboard beam. M receives orders to take station 10.0 miles bearing 060° from the Guide, to pass through a relative point 25.0 miles bearing 085° from the Guide while shifting stations, and to complete the maneuver in 5.0 hours, using minimum speed.

Required.—(a) Courses for M . (b) Speed of M . (c) Required time to reach intermediate position. (See fig. 32.)

Procedure.—Plot the Guide at any convenient point G , and locate the initial, the intermediate, and the final positions of the Maneuvering Unit at M_1 , M_2 , and M_3 . Join M_1 and M_2 , M_2 and M_3 , and M_1 and M_3 .

Lay off $e \dots g$, the vector of the Guide. Transfer the slope $M_1 \dots M_2$ to g and mark it slope (1). Transfer the slope $M_2 \dots M_3$ to g and mark it slope (2).

The distance $M_1 \dots M_3$ will be travelled by the Fictitious Ship in the 5.0 hours allotted. Therefore the Relative Speed of F is equal to $M_1 \dots M_3$ divided by 5, and is plotted as $g \dots f$, parallel to the slope of $M_1 \dots M_3$. The course and speed of F are represented by vector $e \dots f$.

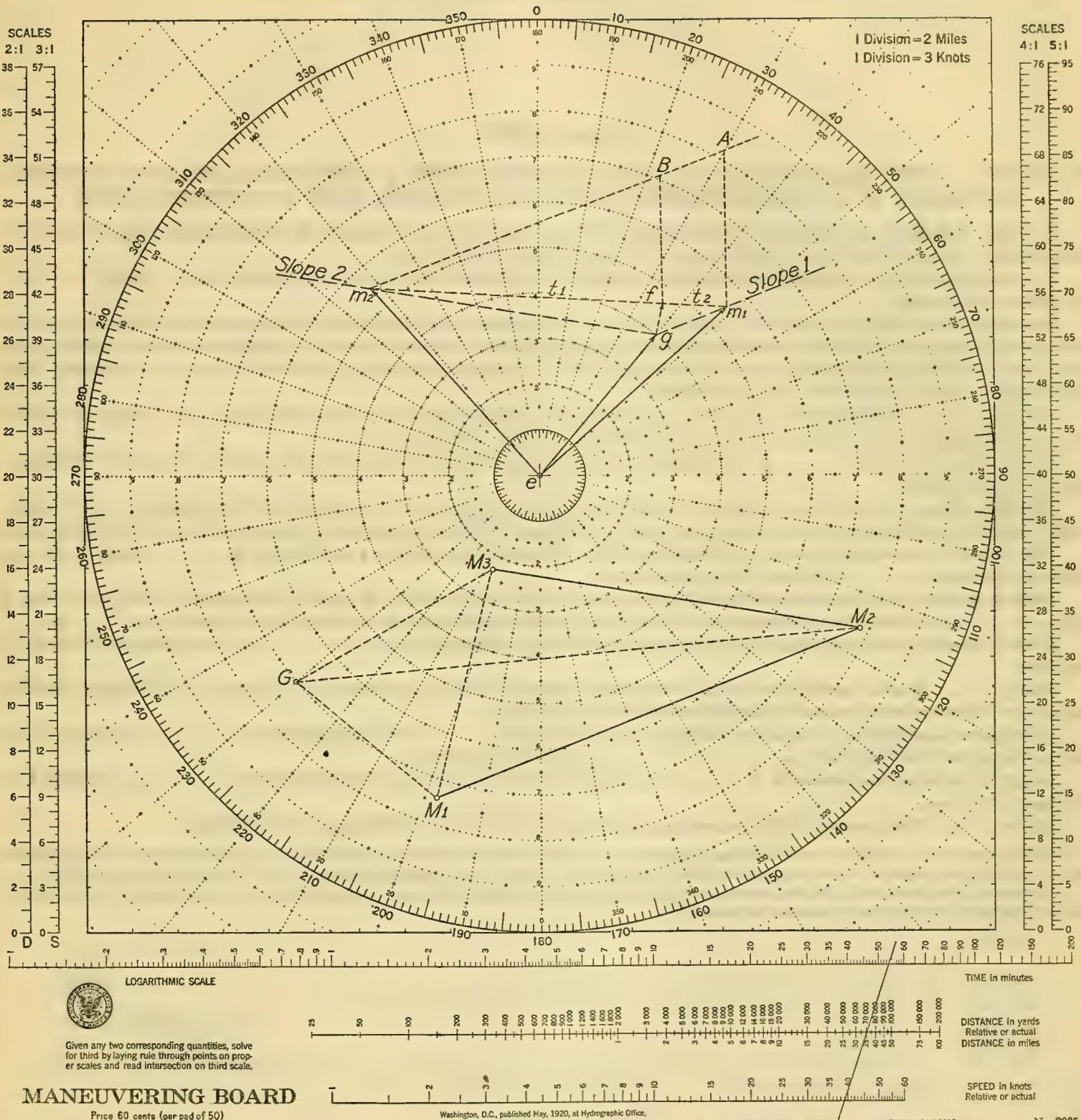
Pivot a straight edge at f , and so orient it that it cuts slope (1) and slope (2) at m_1 and m_2 respectively, both of these points of intersection being equidistant from e . $e \dots m_1$ is the first course for M and $e \dots m_2$ is the course for the second leg. Speed is indicated by either $e \dots m_1$ or $e \dots m_2$.

To obtain the time for M to reach the intermediate position, the Relative Distance $M_1 \dots M_2$ is divided by the Relative Speed $g \dots m_1$. Another way to obtain this time is by use of the Time Line $m_1 \dots f \dots m_2$. Time on first leg is equal to $\frac{f \dots m_2}{m_1 \dots m_2}$ times 5.0 hours.

Answer.—(a) First course $048\frac{1}{2}^\circ$; second course 318° . (b) Speed 16.6 knots. (c) 4.1 hours.

NOTE.—Since the minimum speed was specified, speed must be the same on both legs. The course of speed for M on either leg could have been specified, in which case the corresponding speed or course would have been found by the intersection of the specified course line or speed circle with the slope for that leg.

A rapid method for finding the time on the first leg is to draw any line $m_2 \dots A$ from m_2 equal in length to the total time on both courses, as measured on any convenient scale. A and m_1 are connected and $f \dots B$ is drawn from f parallel to $m_1 \dots A$. $m_2 \dots B$ then measures to the same scale previously chosen the length of time on the first leg.



Given any two corresponding quantities, solve
for third by laying rule through points on prop-
er scales and read intersection on third scale.



Case XXIII

TO SCOUT A GIVEN RELATIVE LINE AS FAR AS POSSIBLE FROM A GIVEN INITIAL RELATIVE POSITION, RETURNING TO ANOTHER RELATIVE POSITION IN SPECIFIED TIME AT SPECIFIED SPEED

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL AND FINAL RELATIVE POSITIONS, DIRECTION OF RELATIVE LINE, SPEED OF SCOUT, AND TOTAL TIME INTERVAL.

TO DETERMINE: COURSES OF SCOUT, LENGTH OF RELATIVE LINE COVERED, AND TIME TO TURN TO FINAL COURSE.

Example.—Fleet is on course 228° , speed 15.0 knots. *Lexington*, now located 85.0 miles due west from the Guide, receives orders to launch a plane at 1200 for purposes of scouting a relative line 180° from the *Lexington* as far as possible, using air speeds of 90.0 knots and landing on the *Saratoga* at 1700. *Saratoga* is stationed 4.0 miles 42° abaft the starboard beam of the Guide. Wind is from 120° , force 20.0 knots.

Required.—(a) Air courses for the plane. (b) Time plane changes course to head for *Saratoga*. (c) Length of relative line covered by plane. (d) Length of chart line covered on first leg. (See fig. 33.)

Procedure.—Lay out $e \dots g$, the course and speed of the Guide, which is also the course and speed of *Saratoga*. Lay out wind's vector $e \dots w$.

Plot Fleet Guide at any convenient point, *G*, and locate relative positions of *Lexington* and *Saratoga* at *L* and *S* respectively. Lay out first Relative Line to be run by plane in direction 180° from *L* of indefinite length and transfer this slope to *g*.

Join *L* and *S*, the distance run by the Fictitious Ship. Transfer this slope to *g* and lay out $g \dots f$ equal to the rate found by dividing *L* . . . *S* by the 5.0 hours available.

With *w* as center and radius equal to 90.0 knots, draw a circle of the plane's air speed, intersecting the first slope from *g* at *p*₁. $w \dots p$ ₁ is the first air course of the plane, which makes good a course indicated on the chart by $e \dots p$ ₁.

Draw a time line from *p*₁ through *f* to the plane's speed circle, intersecting at *p*₂. $w \dots p$ ₂ is the second air course for the plane.

Join *p*₂ . . . *g* and transfer slope to *S*, intersecting outgoing relative line from *L* at *P*, which is the Relative Position reached by the plane when course is changed to head for *Saratoga*.

Time plane is on first leg is found by dividing Relative Distance *L* . . . *P* by Relative Speed $s \dots p$ ₁, which time may also be found from the Time Line *p*₁ . . . *f* . . . *p*₂ as previously explained.

Length of relative line covered is *L* . . . *P*. Length of chart line covered is Ground Speed, $e \dots p$ ₁, multiplied by the time on first leg.

Answer.—(a) First course $176\frac{1}{2}^\circ$; second course $031\frac{1}{2}^\circ$. (b) 1451. (c) 198 miles. (d) 230 miles.

NOTE.—Solution for surface ships is the same except that wind vector and air speed circle are not used. The required speed circles are drawn about *e*, which is the origin of the courses to be steered. The speed on both legs need not be the same, but the Time Line must pass through *f*.

The action of the wind results in the necessity of steering a course to the left of the ground course while on the first leg and the steering of a course to the right of the ground course on the second leg.

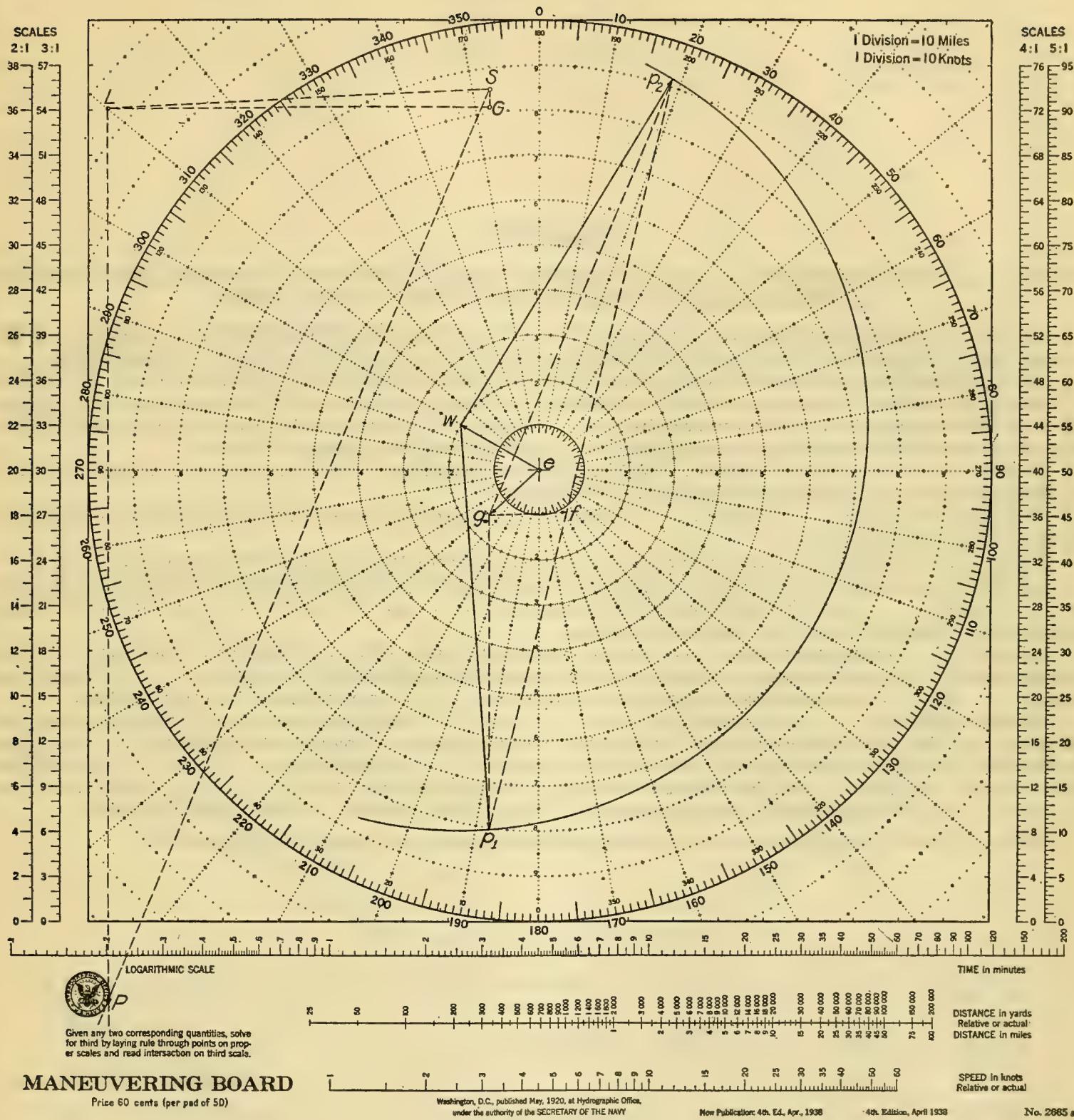


FIGURE 33.

Case XXIV

TO SCOUT A GIVEN CHART LINE AS FAR AS POSSIBLE FROM A GIVEN INITIAL POSITION, RETURNING TO ANOTHER RELATIVE POINT IN GIVEN TIME AT GIVEN SPEED

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL AND FINAL RELATIVE POSITIONS, DIRECTION OF CHART LINE TO BE SCOUTED, SPEED OR SPEEDS OF SCOUT, AND TOTAL TIME OF OPERATION.

TO DETERMINE: COURSES OF SCOUT, LENGTH OF CHART LINE SCOUTED, AND TIME TO TURN.

Example.—Carrier C, now on course 500° at a speed of 20.0 knots, is 60.0 miles, 110° from an air station. A plane takes off from the air station at 0600 to scout a chart line in direction 140° as far as possible, landing on the carrier at 1000, using an air speed of 85.0 knots for the entire period. The true wind is from 340° , velocity 25.0 knots.

Required.—(a) Air courses for the plane. (b) Length of chart line scouted. (c) Time plane turns to head for carrier. (d) Bearing and distance of carrier from plane at (c). (See fig. 34.)

Procedure.—Lay out $e \dots c$ and $e \dots w$, vectors of carrier and wind, respectively. With w as center, draw plane's air speed circle, radius 85.0 knots.

Plot the air station at any convenient point, AS, and locate the present relative position of the carrier at C.

Lay off first chart course of plane from e in the specified direction, 140° , intersecting the plane's speed circle at p_1 . Join $w \dots p_1$ and $c \dots p_1$. The first air course for the plane is $w \dots p_1$. The vector $c \dots p_1$ indicates the slope of the Relative Movement Line of the plane on its first leg.

Join AS \dots C and transfer slope to c . Along this slope lay off rate AS \dots C divided by total time 4.0 hours, locating point f. The Relative Speed and course of the Fictitious Ship is indicated by this vector $c \dots f$. Draw Time Line $p_1 \dots f \dots p_2$, intersecting the plane's speed circle at p_2 . $w \dots p_2$ is the second air course of plane.

Transfer the slope $c \dots p_1$ to AS and the slope $c \dots p_2$ to C. These slopes intersect at P, the turning point for the plane. The time on the first leg is found by dividing the Relative Distance AS by the Relative Speed $c \dots p_1$ or else by using the Time Line and the formula. Time on first leg equals $f \dots p_2$ divided by $p_1 \dots f \dots p_2$ times Total Time Allotted. The length of chart line covered is found by multiplying the Ground Speed, $e \dots p_1$, by time on first leg.

Answer.—(a) First course $134\frac{1}{2}^\circ$; second course 358° . (b) 162 miles. (c) 0729. (d) 349° distant 123 miles.

NOTE.—To illustrate the scouting of a line in a given chart direction by a surface ship, example XXIV-B is appended.

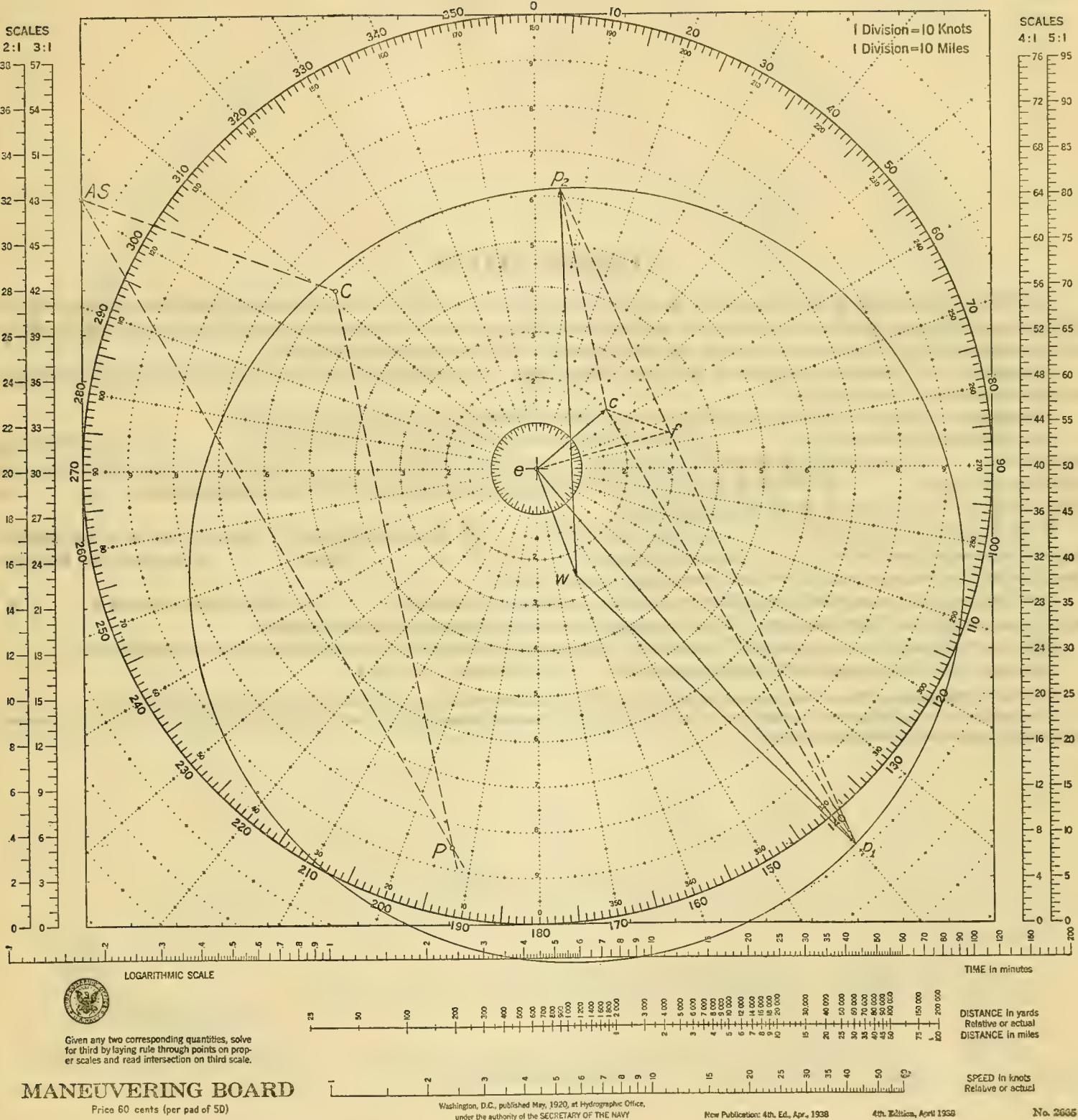


FIGURE 34.

Example XXIV-B

GIVEN: Fleet Guide is on course 340° , speed 16.0 knots, and at 0800 is 20.0 miles bearing 250° from chart point A . At this time a destroyer leaves point A to scout a chart line in direction 000° to maximum distance so as to join the Guide at 1300, using a speed of 25.0 knots on the first leg and a speed of 20.0 knots on the second leg.

Required.—(a) Destroyer's course on first and second legs. (b) Length of chart line scouted. (c) Time Destroyer turns to second leg. (See Fig. 35.)

Procedure.—Locate the chart point at any point, A , and the 0800 position of the Guide at G .

Lay out the Guide's vector, $e \dots g$, and the vector of the first leg for the Destroyer, $e \dots d_1$. Lay off from g the vector $g \dots f$, equal in length to the rate obtained by dividing the distance $A \dots G$ by the 5.0 hours available, and parallel to the slope $A \dots G$. From d_1 draw a Time Line through f , intersecting the 20.0 knot speed circle at d_2 . $e \dots d_2$ is the vector for the second leg of the destroyer's run.

From A , in direction parallel to $g \dots d_1$, lay off a line $A \dots X$, of extended length. From G , lay off a line parallel to, but in opposite direction to, $g \dots d_2$, intersecting $A \dots X$ at P . $A \dots P$ and $P \dots G$ represent the Relative Lines run by the destroyer on its first and second legs respectively.

The time on first leg may be found either by dividing the Relative Distance $A \dots P$ by the Relative Speed $g \dots d_1$, or by proper use of the Time Diagram. Time to change to second leg is this time added to 0800.

The chart distance run on course 000° is found by multiplying the speed of 25.0 knots by the time on the first leg.

Answer.—(a) First course 000° ; second course 272° . (b) 67.0 miles. (c) 1041.

NOTE.—A Navigational Plot of this example is shown in half scale for illustrative purposes, but is not required for the solution.

A comparison of this example of surface vessels with the previous example using airplanes, will further emphasize the superiority of aircraft for covering large areas in scouting exercises.

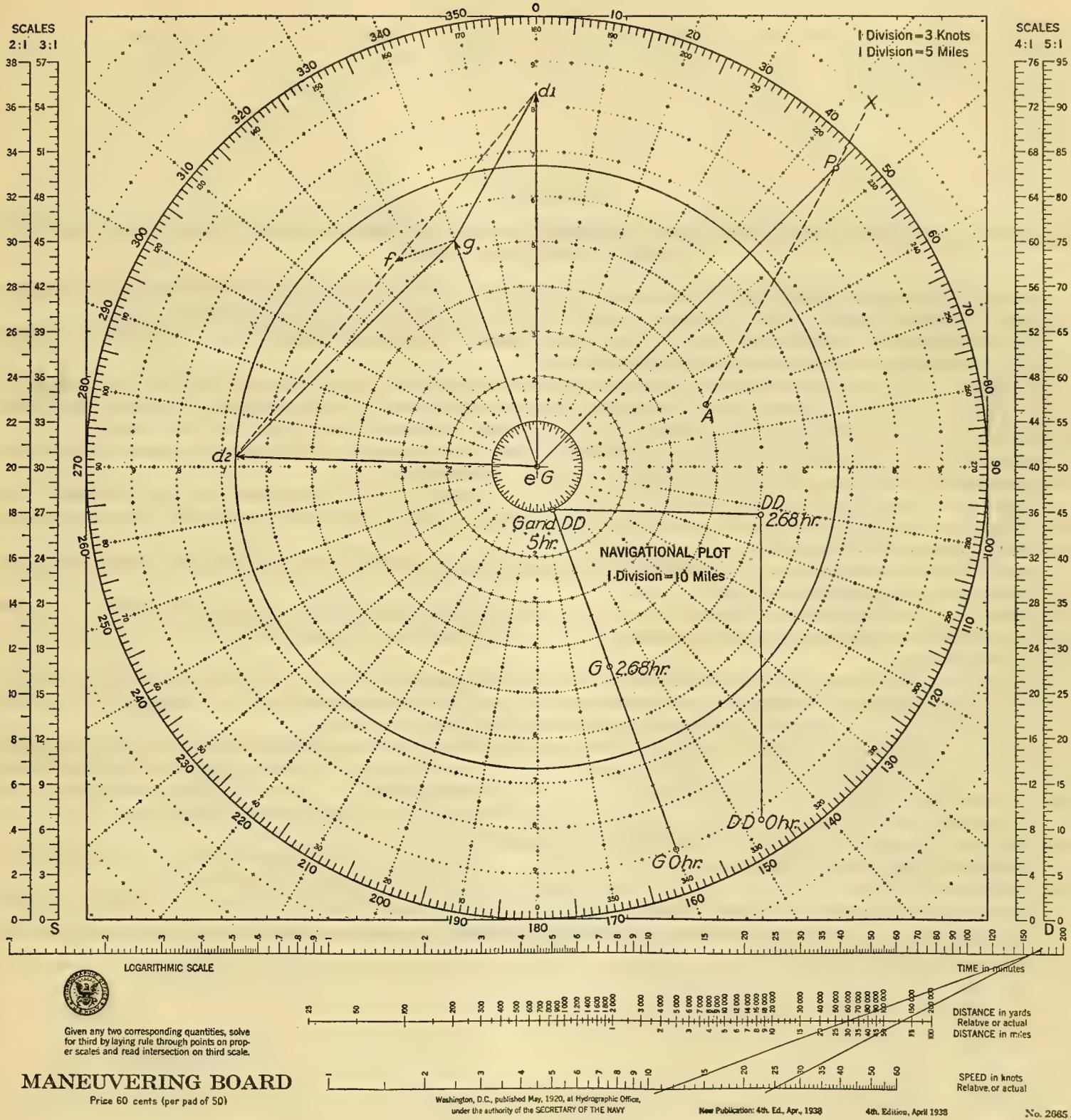


FIGURE 35.

Case XXV

TO SCOUT A GIVEN RELATIVE LINE AS FAR AS POSSIBLE SO AS TO REACH A GIVEN CHART POINT IN GIVEN TIME AT SPECIFIED SPEED

GIVEN: COURSE AND SPEED OF THE GUIDE, INITIAL RELATIVE AND FINAL CHART POSITIONS, DIRECTION OF RELATIVE LINE TO BE SCOUTED, SPEED OF SCOUT, AND TOTAL TIME INTERVAL.

TO DETERMINE: COURSES OF SCOUT, LENGTH OF RELATIVE LINE COVERED, LOCATION OF TURNING POINT, AND TIME OF TURNING TO SECOND COURSE.

Example.—*Lexington*, on course 315° at speed 25.0 knots, at 0615 bears 210° , distant 70.0 miles from a Naval Air Station. At 0615 a plane is launched to scout a relative line in direction 340° from the carrier, as far as possible, returning to the Naval Air Station in 4.0 hours. Plane has air speed 70.0 knots; and the wind is from 100° with a velocity of 22.0 knots.

Required.—(a) Air courses for the plane. (b) Relative length of line covered. (c) Bearing and distance of *Lexington* when plane starts second leg. (d) Time plane starts second leg. (See fig. 36.)

Procedure.—Lay out vectors of *Lexington* and wind, $e \dots I$ and $e \dots w$, respectively, and draw 70.0 knot speed circle with w as center. Plot position of *Lexington* at any point, L , and locate Naval Air Station at N , 70.0 miles bearing 030° from *Lexington*'s 0615 position.

From I , lay out the slope of the required Relative Movement in direction 340° , intersecting the plane's air speed circle at p_1 . The first air course for the plane is $w \dots p_1$.

Draw $L \dots N$, and transfer slope to e , since $L \dots N$ represents an instantaneous chart position at 0615. Lay out along this slope the rate obtained by dividing distance $L \dots N$ by total time, 4.0 hours, locating f . $e \dots f$ is the vector of the Fictitious Ship.

From p_1 draw a Time Line through f , intersecting the plane's speed circle at p_2 . $w \dots p_2$ is the course for the second leg.

Using the 0615 position of *Lexington*, the charted position N , and the courses made good over the ground by the plane, $e \dots p_1$ and $e \dots p_2$, the chart track of the plane is from L to P and back to N .

Maintaining L as the fixed position of the Guide, the position of N must be offset an amount equal to the Relative Movement of N during the 4.0 hours allowed. This is in direction $I \dots e$ and locates N' , the position of the Naval Air Station relative to *Lexington* at 1015. Transfer the slope of $I \dots p_1$ to L and the slope of $I \dots p_2$ to N' . These two slopes intersect at T , the turning point for the plane. $L \dots T$ is the relative length of line covered before changing course. $T \dots L$ is the bearing and distance of *Lexington* from plane at turning point.

Time on first leg is found by dividing $L \dots T$ by rate $I \dots p_1$, $L \dots P$ by ground speed $e \dots p_1$, or by use of the proportions of the Time Line $p_1 \dots f \dots p_2$. This time added to 0615 gives the time to turn.

Answer.—(a) First course 347° ; second course $114\frac{1}{2}^\circ$. (b) 95.5 miles. (c) Bearing 160° , distant 95.5 miles. (d) 0755.

NOTE.—The solution of this example would be the same for a surface scout except that the courses would be referred to e instead of w . Also speeds need not be the same on each leg.

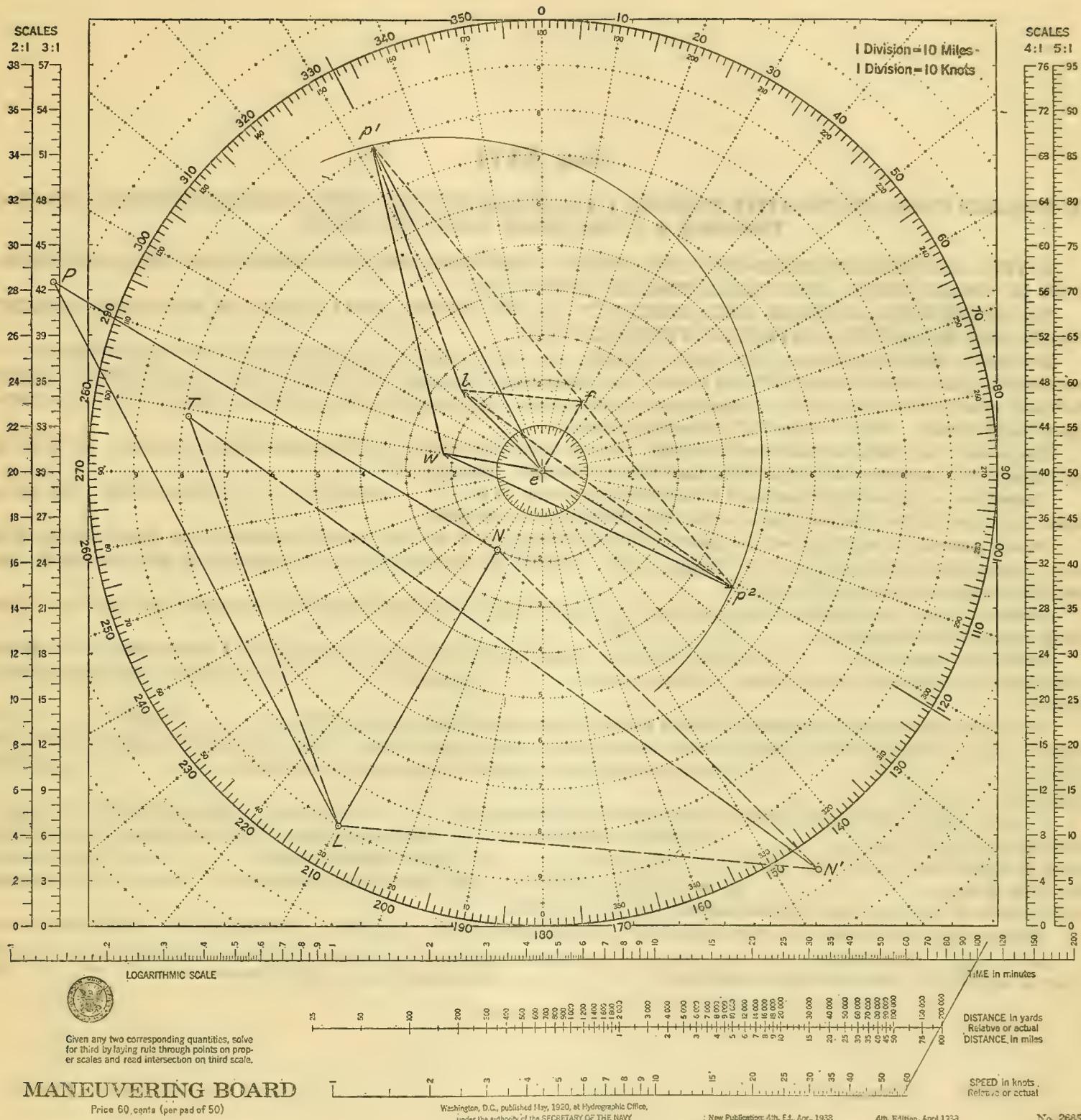


FIGURE 36.

Case XXVI

TO PROCEED FROM ONE RELATIVE POSITION TO ANOTHER IN GIVEN TIME AT MINIMUM SPEED, PASSING THROUGH A GIVEN CHART POINT EN ROUTE

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL AND FINAL RELATIVE POSITIONS, RELATIVE LOCATION OF CHART POINT, AND TOTAL TIME OF OPERATION.

TO DETERMINE: COURSES AND SPEEDS OF MANEUVERING UNIT, TIME OF REACHING CHART POINT, AND RELATIVE POSITION OF TURNING POINT.

Example.—Fleet is on course 140° , speed 12.0 knots. At 0630 a plane is launched from the *Saratoga* with orders to investigate a chart point bearing 200° and distant 105.0 miles from *Saratoga* and to land on the *Ranger*, which is located 45.0 miles ahead of *Saratoga*, and is on fleet course at fleet speed. The plane is to complete the maneuver at 0930, using minimum speed en route. Wind is from 010° , velocity 21.0 knots.

Required.—(a) Minimum air speed for plane. (b) Air courses for plane. (c) Time chart point is reached. (d) Bearing and distance of plane from *Saratoga* when chart point is reached. (See fig. 37.)

Procedure.—Lay out the wind's and the *Saratoga*'s vectors as $e \dots w$ and $e \dots s$, respectively. The latter is also the vector of the *Ranger*, which is also using Fleet Course and speed.

At any convenient point lay out the position of the *Saratoga* at S , and the relative position of the chart point at 0630 at P . Locate the initial position of the *Ranger* at R . Advance the position of R to R' , representing 0930 position of this ship, 36.0 miles ahead of the 0630 position.

Connect S and P and transfer this slope to e . In a similar manner, connect P and R' and transfer this slope to e . The plane travels the chart track $S \dots P \dots R'$.

Divide the distance $S \dots R$ by the 3.0 hours allowed, giving the rate travelled by the Fictitious Ship in its direct route from the *Saratoga* to the *Ranger*. From s , draw $s \dots f$, parallel to $S \dots R$ and equal to the rate obtained above.

Pivot a straight-edge at f and orient it until it cuts the slopes drawn from e and parallel to $S \dots P$ and $P \dots R'$, respectively, at points equidistant from w , these points being p_1 and p_2 , respectively. The air speed for the plane is given by either $w \dots p_1$ or $w \dots p_2$. The first course for the plane is indicated by the direction of $w \dots p_1$ and the second air course by $w \dots p_2$.

From S , draw a Relative Movement Line parallel to the slope of $s \dots p_1$ and from R , draw a Relative Movement Line parallel to the slope of $p_2 \dots s$. These two lines intersect at P'' , the turning point. The time required to reach this point is either found by dividing the Relative Distance $S \dots P''$ by the Relative Speed $s \dots p_1$ or by the use of the Time Line $p_1 \dots f \dots p_2$, as previously explained. The bearing and distance of P'' from S is the bearing and distance of the plane from the *Saratoga* at the turning point.

Answer.—(a) 66.8 knots. (b) First course 203° ; second course $051\frac{1}{2}^\circ$. (c) 0740. (d) Bearing 207° , distant 96.0 miles.

NOTE.—Except for the influence of the wind, this case is most rapidly solved by the Navigational Plot, the method used for surface vessels. It will be noted that the Navigational plot is shown by $S \dots P \dots R'$, while the Relative Plot is shown by $S \dots P'' \dots R$.

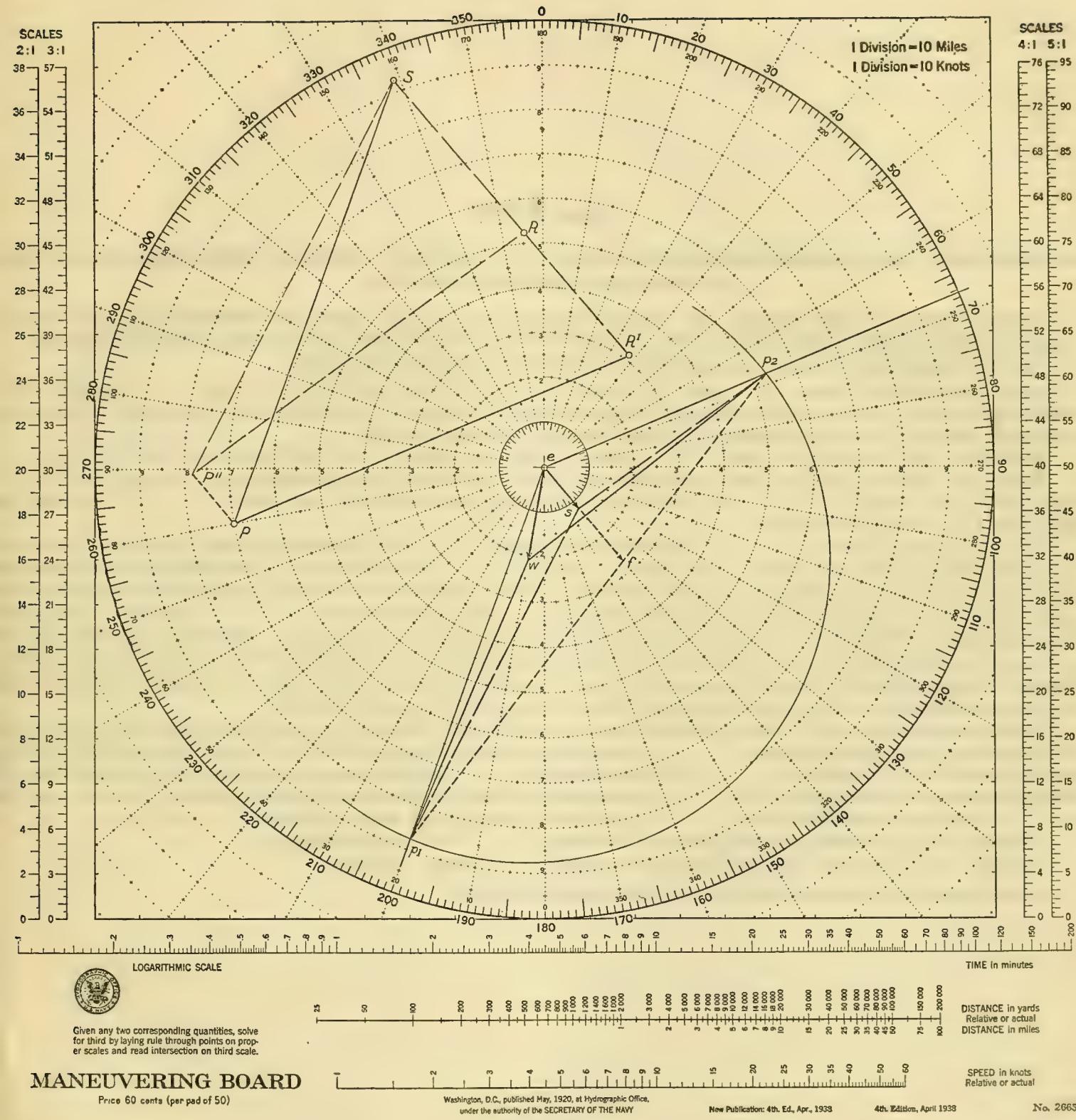


FIGURE 37.

Case XXVII

TO SCOUT OUT AND IN ALONG GIVEN RELATIVE LINE AT MINIMUM SPEED IN GIVEN TIME, RETURNING TO ORIGINAL RELATIVE POSITION

GIVEN: COURSE AND SPEED OF GUIDE, INITIAL RELATIVE POSITION (WHICH IS ALSO FINAL RELATIVE POSITION), DIRECTION AND LENGTH OF RELATIVE LINE, AND TOTAL TIME INTERVAL.

TO DETERMINE: COURSES AND SPEED OF SCOUT AND TIME TO TURN.

Example—Guide on course 320° , speed 12.0 knots, has a scout stationed 4.0 miles broad on her starboard bow. Scout receives orders to scout, at minimum speed, to a distance of 25.0 miles from her present station, maintaining constant bearing on the Guide, leaving formation at 1300 and returning to station at 1600.

Required.—(a) Speed used by Scout. (b) Courses for Scout. (c) Time to turn to second course. (See fig. 38.)

Procedure.—Plot the Guide at point *G*, and locate the initial position and the intermediate point of the scout at *S* and *P* respectively. Join *S* and *P*. *S . . . P* is the Relative Movement Line on the first course and *P . . . S* is the Relative Movement Line for the second course.

Lay out the Guide's vector, *e . . . g*, and transfer the slope *S . . . P* to *g*, extending in both directions. The use of the regular Fictitious Ship in this case would involve a vector *g . . . f* of length equal to zero, since the initial and the final positions coincide, and yielding an indeterminate result. This is overcome by introducing a second Fictitious Ship whose vector is equal to the length of the perpendicular erected from *e* to the transferred slope *S . . . P*. Relative to this extra Fictitious Ship the Fictitious Scout travels at the same rate both-going out and returning. Mathematically this constant rate is equal to the Relative Distance divided by the total time available plus the square root of the sum of the squares of the Relative Distance divided by the total time available and of the Relative Speed of the Guide to the Second Fictitious Ship. This process is laborious, so the procedure outlined below will be used as the graphic solution.

Erect a perpendicular at *e* to the transferred slope, cutting it at *O*. Extend this perpendicular beyond the slope line. Along this same perpendicular lay out *O . . . Q* equal to the Relative Distance divided by the total time available and to the speed scale in use. Connect *Q . . . g* and lay out length *Q . . . g* from *Q* along the perpendicular, locating point *R*. With *O* as center and radius equal to *O . . . R*, swing an arc cutting the transferred slope at *s₁* and *s₂*. The first course is *e . . . s₁* and the second course is *e . . . s₂*. The speed is shown by the length of either *e . . . s₁* or *e . . . s₂*.

Time on first leg is found by dividing Relative Distance *S . . . P* by Relative Speed *g . . . s₁*.

Answer.—(a) 22.0 knots. (b) First course 342° ; second course $207\frac{1}{2}^\circ$. (c) 1508.

NOTE.—The solution of this case using aircraft is shown in example XXVII-B.

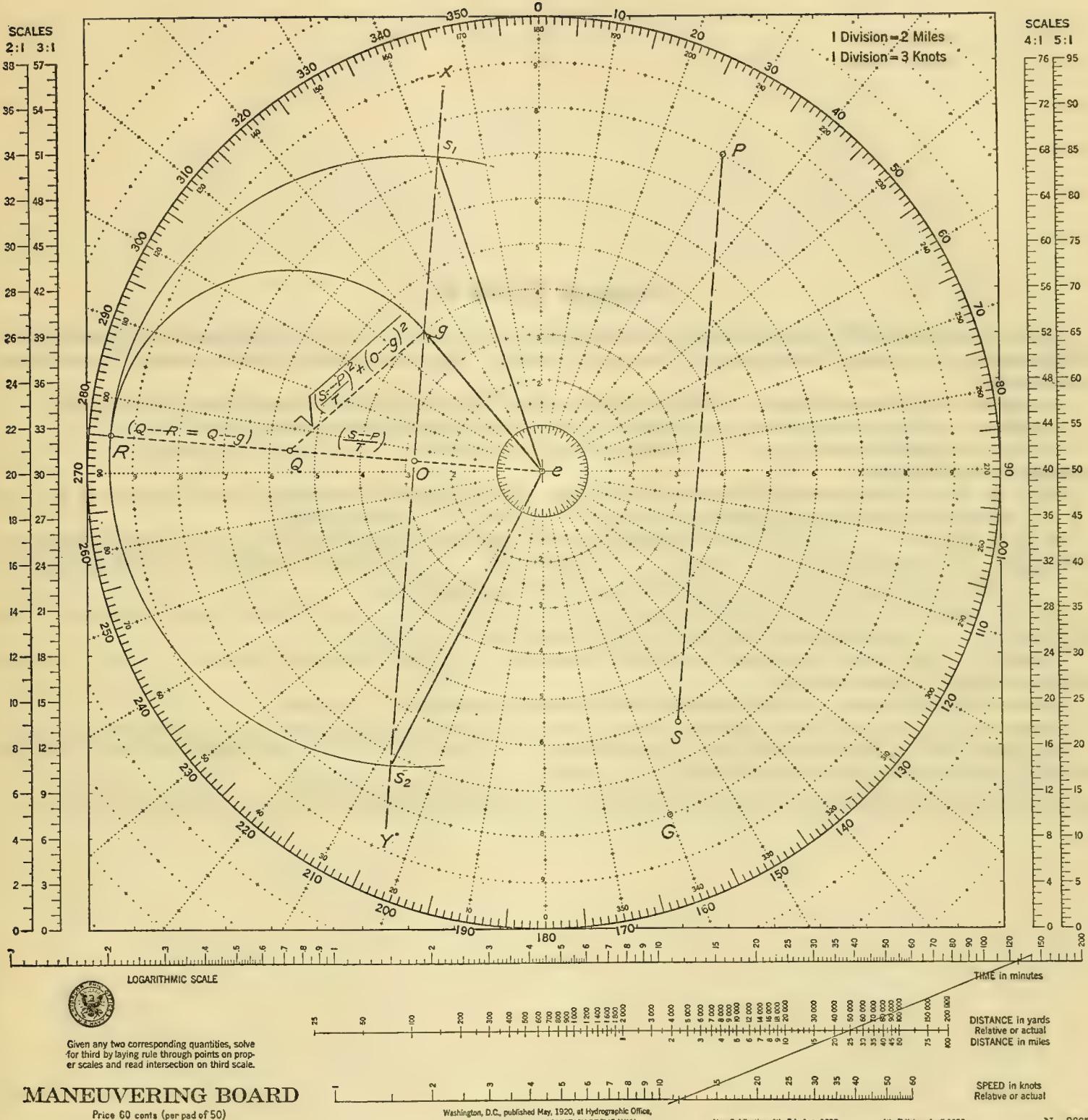


FIGURE 38.

Example XXVII-B

Carrier C, on course 140° , speed 20.0 knots, launches a plane at 1115 to scout a relative point bearing 210° , distant 100.0 miles, returning in 2.5 hours. Wind is from 170° , velocity 25.0 knots. The maneuvers to the Carrier to launch the plane may be disregarded.

Required.—(a) Air speed of plane. (b) Air courses for plane. (c) Time to turn to homing course. (See fig. 39.)

Procedure.—Plot position of Carrier at any point C, and the relative point to be reached by the plane at P.

Lay out the Carrier's vector at e c and the wind's vector at e w. Transfer the slope C P to c, extending in both directions.

Since the wind is the controlling factor with all aircraft problems, erect a perpendicular to the transferred slope of C P from w, intersecting the transferred slope at O. Extend w O indefinitely.

From O, lay out O Q equal to the Relative Distance, 100.0 miles, divided by the 2.5 hours available, and to the speed scale in use. Join Q and C and lay out Q R equal to Q C in length. With O as center and radius equal to O R, swing an arc cutting the transferred slope at p_1 and p_2 , respectively.

The air speed of the plane is given by the length of w p_1 or w p_2 . First air course is indicated by the direction of w p_1 and the second air course by the direction as w p_2 .

Time on first leg is found by dividing the Relative Distance C P by the Relative Speed c p_1 . This time added to 1115 gives the time to turn.

Answer.—(a) 95.0 knots. (b) First air course $188\frac{1}{2}^\circ$; second air course 051° . (c) 1252.

NOTE.—It is immaterial whether the perpendicular extends toward e or away from e as the same points p_1 and p_2 will result in either case.

The ground courses and speeds of the plane are indicated by vectors e p_1 and e p_2 . If it were required that a plane guard trail the plane, it would therefore take the courses indicated by e p_1 and e p_2 .

In case the transferred slope passes through e for surface vessels or w for aircraft, the perpendicular from e or w respectively would have zero length, but it would still have direction. The procedure outlined above is still carried out, considering that the point O would coincide with e or w.

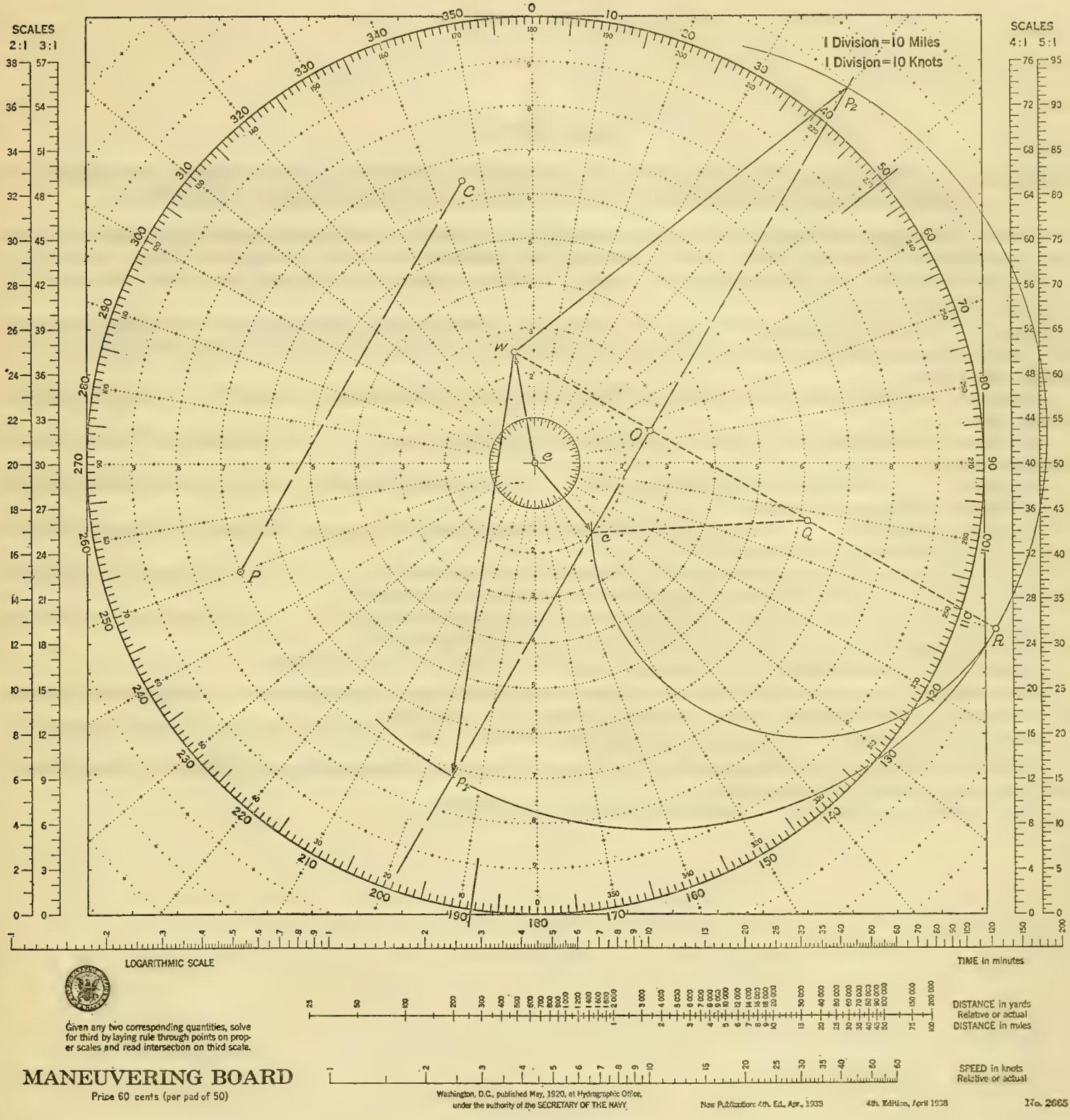


FIGURE 39.

Case XXVIII

TO SCOUT IN A GIVEN RELATIVE DIRECTION FOR MAXIMUM DISTANCE AT GIVEN SPEED, RETURNING TO ORIGINAL RELATIVE POSITION IN GIVEN TIME, USING GIVEN SPEED (OR SPEEDS)

GIVEN: COURSE AND SPEED OF GUIDE, COINCIDENT INITIAL AND FINAL RELATIVE POSITIONS OF SCOUT, DIRECTION OF RELATIVE LINE TO BE SCOUTED, SPEED OF SCOUT, AND TOTAL TIME INTERVAL.

TO DETERMINE: COURSES OF SCOUT, TIME TO TURN TO SECOND COURSE, AND LENGTH OF RELATIVE LINE SCOUTED.

Example.—A Carrier is on course 090° , speed 17.5 knots, and at 0600 launches a plane to scout a relative line in direction 150° from the Carrier, as far as possible at an air speed of 85.0 knots, returning to the Carrier at 0820. Wind is from 110° , velocity 18.0 knots.

Required.—(a) Air courses for the plane. (b) Time to turn to incoming course. (c) Length of relative line covered. (See fig. 40.)

Procedure.—Plot Carrier at any point, C, and lay out the Relative Line C X in direction 150° from C and of indefinite length.

Lay out e c, the vector of the Carrier, and e w, the vector of the wind. With w as center and the airspeed of the plane as radius, inscribe a circle.

Transfer the slope of C X to c, cutting the plane's airspeed circle at p_1 and p_2 . The plane's outgoing air course is w p_1 and the incoming air course is w p_2 .

The time on the first course is either found by means of the Time Line $p_1 c p_2$ or else by the graphical method explained in case XXI. This time is added to 0600 to find the time to turn.

The length of Relative Line scouted is found by multiplying the Relative Speed, $c p_1$, by the time on the first leg or the Relative Speed, $c p_2$, by the time on the second leg, and drawn as C P.

Answer.—(a) First air course $131\frac{1}{2}^\circ$; second air course 348° . (b) 0730. (c) 87.0 miles.

NOTE.—With both the direction of relative movement and the plane's airspeed given, it is a simple matter to find the desired air courses. The utilization of the Time Line provides a ready solution for the time on first and second legs.

As in case XXVII, the Fictitious Ship's vector coincides with the Guide's vector since the initial and final relative positions of the scout are the same. The line $p_1 c p_2$ is therefore a Time Line.

The Relative Plot is not required if the scout leaves and returns to the Guide, but it is necessary otherwise if it is desired to know the scout's position relative to the Guide at the turning point.

In case the scout is a surface vessel, the speed circles are drawn from e instead of w.

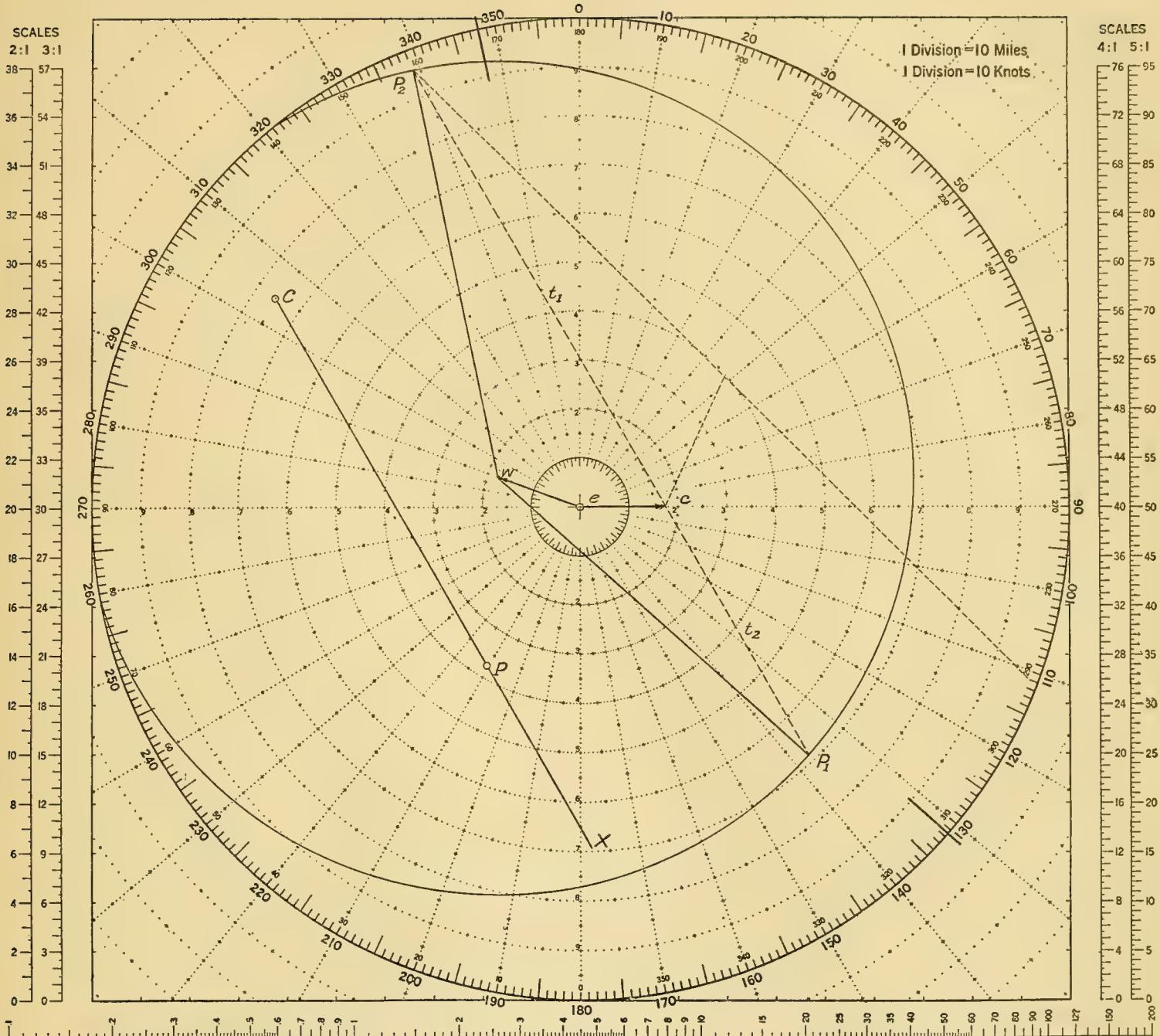


FIGURE 40.

